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UNITS AND PHYSICAL CONSTANTS.

UNITS AND

PHYSICAL CONSTANTS.

UNIVERSITY OF CALIFORNIA
DEPARTMENT OF PHYSICS

BY

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SECOND EDITION.

London:
MACMILLAN AND CO.,
AND NEW YORK.
1886.

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QC61
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GLASGOW: PRINTED AT THE UNIVERSITY PRESS
BY ROBERT MACLEHOSE.

PREFACE TO "ILLUSTRATIONS OF THE C.G.S. SYSTEM OF UNITS," PUBLISHED BY THE PHYSICAL SOCIETY OF LONDON IN 1875.

THE quantitative study of physics, and especially of the relations between different branches of physics, is every day receiving more attention.

To facilitate this study, by exemplifying the use of a system of units fitted for placing such relations in the clearest light, is the main object of the present treatise.

A complete account is given of the theory of units *ab initio*. The Centimetre-Gramme-Second (or C.G.S.) system is then explained; and the remainder of the work is occupied with illustrations of its application to various branches of physics. As a means to this end, the most important experimental data relating to each subject are concisely presented on *one uniform scale* — a luxury hitherto unknown to the scientific calculator.

I am indebted to several friends for assistance in special departments—but especially to Professor Clerk Maxwell and Professor G. C. Foster, who revised the entire manuscript of the work in its original form.

Great pains have been taken to make the work correct as a book of reference. Readers who may discover any errors will greatly oblige me by pointing them out.

PREFACE TO FIRST EDITION OF UNITS AND PHYSICAL CONSTANTS, 1879.

THIS Book is substantially a new edition of my "Illustrations of the C.G.S. System of Units" published in 1875 by the Physical Society of London, supplemented by an extensive collection of physical data. The title has been changed with the view of rendering it more generally intelligible.

Additional explanations have been given upon some points of theory, especially in connection with Stress and Strain, and with Coefficients of Diffusion. Under the former head, I have ventured to introduce the terms "resilience" and "coefficient of resilience," in order to avoid the multiplicity of meanings which have become attached to the word "elasticity."

A still greater innovation has been introduced in an extended use of the symbols and processes of multiplication and division, in connection with equations which express not numerical but physical equality. The advantages of this mode of procedure are illustrated by its application to the solution of the most difficult problems on units that I have been able to collect from standard text-books (chiefly from Wormell's 'Dynamics').

. . . . . .

I am indebted to several friends for contributions of experimental data.

A Dutch translation of the first edition of this work has been made by DR. C. J. MATTHES, Secretary of the Royal Academy of Sciences of Amsterdam, and was published in that city in 1877.

Though the publication is no longer officially connected with the Physical Society, the present enlarged edition is issued with the Society's full consent and approval.

PREFACE TO THE PRESENT EDITION.

In collecting materials for this edition, I have gone carefully through the Transactions and Proceedings of the Royal Society, the Royal Society of Edinburgh, and the Physical Society of London, from 1879 onwards, besides consulting numerous papers, both English and foreign, which have been sent to me by their authors. I have also had the advantage of the co-operation of Dr. Pierre Chappuis (of the *Bureau International des Poids et Mesures*), who has for some years been engaged in preparing a German edition. Several items have been extracted from the very elaborate and valuable *Physikalisch-Chemische Tabellen* of Landolt and Börnstein (Julius Springer, Berlin, 1883). Among friends to whom I am indebted for data or useful suggestions, are Prof. Barrett, Mr. J. T. Bottomley, Prof. G. C. Foster, Prof. Lodge, Prof. Newcomb, Mr. Preece, and the Astronomer-Royal.

The expository portions of the book are for the most part unchanged; but a Supplemental Section has been added (p. 34) on physical deductions from the dimensions of units; a simplification has been introduced in the discussion of adiabatic compression (p. 125); and the account of thermoelectricity (p. 172) has been re-written and enlarged. The name "thermoelectric height" has been introduced to denote the element usually represented by the ordinates of a thermoelectric diagram.

The preliminary "Tables for reducing other measures to C.G.S. measures" have been greatly extended, and in each case the reciprocal factors are given which serve for reducing from C.G.S. measures to other measures. Professor Miller's comparison of the kilogramme and pound is supplemented by three later comparisons officially made at the *Bureau International.*

A nearly complete list of the changes and additions now introduced is appended to this Preface, as it will probably be useful to possessors of the previous edition.

The adoption of the Centimetre, Gramme, and Second, as the fundamental units, by the International Congress of Electricians at Paris in 1881, led to the immediate execution of a French translation of this work, which was published at Paris by Gauthier-Villars in 1883. The German translation was commenced about the same time, but the desire to perfect its collection of physical data has caused much delay. It will be brought out by Ambrosius Barth, the publisher of Wiedemann's *Annalen.* A Polish edition, by Prof. J. J. Boguski, was published at Warsaw in 1885; and permission has been asked and granted for the publication of an Italian edition.

J. D. EVERETT.

BELFAST, *September*, 1886.

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UNITS AND PHYSICAL CONSTANTS.

TABLES FOR REDUCING TO AND FROM C.G.S. MEASURES.

The abbreviation *cm.* is used for *centimetre* or *centimetres*,
gm. „ *gramme* or *grammes*,
c.c. „ *cubic centimetre(s)*.

The numbers headed "reciprocals" are the factors for reducing *from* C.G.S. measures.

Length.

	cm.	Reciprocals.
1 inch,	= 2·5400	·39370
1 foot,	= 30·4797	·032809
1 yard,	= 91·4392	·010936
1 mile,	= 160933	$6{\cdot}2138 \times 10^{-6}$
1 nautical mile,	= 185230	$5{\cdot}398 \times 10^{-6}$

More exactly, according to Captain Clarke's comparisons of standards of length (printed in 1866), the metre is equal to 1·09362311 yard, or 3·2808693 feet, or 39·370432 inches, the standard metre being taken as correct at 0° C., and the standard yard as correct at $16\frac{2}{3}$° C. Hence the inch is 2·5399772 centimetres.

Area.

	sq. cm.	Reciprocals.
1 sq. inch,	= 6·4516	·1550
1 sq. foot,	= 929·01	·001076
1 sq. yard,	= 8361·13	·0001196
1 sq. mile,	= $2{\cdot}59 \times 10^{10}$	$3{\cdot}861 \times 10^{-11}$

Volume.

	cub. cm.	Reciprocals.
1 cubic inch, -	= 16·387	·06102
1 cubic foot, -	= 28316·	$3·532 \times 10^{-5}$
1 cubic yard, -	= 764535·	$1·308 \times 10^{-6}$
1 pint, - - -	= 567·63	·001762
1 gallon, - - -	= 4541·	·0002202

Mass.

	gm.	Reciprocals.
1 grain, - -	= ·0647990	15·432
1 ounce avoir., -	= 28·3495	·035274
1 pound ,, -	= 453·59	·0022046
1 ton, - - -	= $1·01605 \times 10^6$	$9·84206 \times 10^{-7}$

According to the comparison made by Professor W. H. Miller in 1844 of the "kilogramme des Archives," the standard of French weights, with two English pounds of platinum, and additional weights, also of platinum, the kilogramme is 15432·34874 grains, of which the new standard pound contains 7000. Hence the kilogramme would be 2·2046212 pounds, and the pound 453·59265 grammes.

Three standard pounds, one of platinum-iridium and the other two of gilded bronze, belonging to the *Standards Department*, were compared, in 1883, at the *Bureau International des Poids et Mesures*, with standards belonging to the *Bureau*, and their values in grammes were found to be respectively

453·59135,
453·58924,
453·58738.

—*Travaux et Mémoires*, tome IV.

Velocity.

	cm. per sec.	Reciprocals.
1 foot per second, - -	= 30·4797	·032809
1 statute mile per hour, -	= 44·704	·022369
1 nautical mile per hour, -	= 51·453	·019435
1 kilometre per hour, -	= 27·777	·036

Acceleration.

	cm. per sec. per sec.	Reciprocal.
1 ft. per sec. per sec., -	= 30·4797	·032809

Density.

	gm. per c.c.	Reciprocals.
Pure water at temperature of maximum density, - -	= 1·000013	·999987
1 lb. per cubic foot, - -	= ·016019	62·426
1 grain per cubic inch, - -	= ·003954	252·88

Stress (in gravitation measure).

	gm. per sq. cm.	Reciprocals.
1 lb. per sq. foot, - =	·48826	2·0481
1 lb. per sq. inch, - =	70·31	·014223
1 inch of mercury at 0° C., - - - =	34·534	·0028957
30 inches ,, ,,	= 1036·0	·00096525
760 mm. ,, ,,	= 1033·3	·00096777

Surface Tension (in gravitation measure).

	gm. per cm.	Reciprocals.
1 grain per linear inch, - =	·02551	39·20
1 lb. ,, foot, -	= 14·88	·06720

Work (in gravitation measure).

	gm.-cm.	Reciprocals.
1 foot-pound, -	= 13825	$7·2331 \times 10^{-5}$
1 foot-grain, -	= 1·975	·50632
1 foot-ton, - -	= $3·097 \times 10^{7}$	$6·494 \times 10^{-8}$
1 kilogrammetre, -	= 10^{5}	10^{-5}

Rate of Working (in gravitation measure).

	gm.-cm. per sec.	Reciprocals.
1 horse-power, -	= $7·604 \times 10^{9}$	$1·3151 \times 10^{-7}$
1 force-de-cheval, -	= $7·5 \times 10^{6}$	$1·3333 \times 10^{-7}$

Heat (in gravitation measure).

	gm.-cm.	Reciprocals.
1 gm. deg., - -	= 42400	$2·36 \times 10^{-5}$
1 lb. deg. Cent., -	= $1·923 \times 10^{7}$	$5·2 \times 10^{-8}$
1 ,, Fahr., -	= $1·068 \times 10^{7}$	$9·36 \times 10^{-8}$

The following reductions of gravitation measures to absolute measures are on the assumption that $g = 981$:—

Force (*in absolute measure*).

	Dynes.	Reciprocals.
Weight of 1 gm., - - =	981	·001019
,, 1 kilogm., - =	$9{\cdot}81 \times 10^5$	$1{\cdot}019 \times 10^{-6}$
,, 1 tonne, - =	$9{\cdot}81 \times 10^8$	$1{\cdot}019 \times 10^{-9}$
,, 1 ton, - - =	$9{\cdot}97 \times 10^8$	$1{\cdot}003 \times 10^{-9}$
,, 1 cwt., - - =	$4{\cdot}98 \times 10^7$	$2{\cdot}008 \times 10^{-8}$
,, 1 lb. avoir., - =	$4{\cdot}45 \times 10^5$	$2{\cdot}247 \times 10^{-6}$
,, 1 oz. ,, - =	$2{\cdot}78 \times 10^4$	$3{\cdot}597 \times 10^{-5}$
,, 1 grain, - =	63·57	·01573
1 poundal, - - -	= 13825	$7{\cdot}2333 \times 10^{-5}$

(The ratio of the poundal to the dyne is independent of g.)

Stress (*in absolute measure*).

	Dynes per sq. cm.	Reciprocals.
1 lb. per sq. foot, - =	·479	·00209
1 lb. ,, inch, - =	$6{\cdot}9 \times 10^4$	$1{\cdot}45 \times 10^{-5}$
1 gm. ,, cm., - =	·981	·00102
1 kilo. ,, decim., - =	$9{\cdot}81 \times 10^3$	$1{\cdot}02 \times 10^{-4}$
1 cm. of mercury at 0° C.,	= 13338·	·0000736
76 ,, ,, ,, =	$1{\cdot}0136 \times 10^6$	$9{\cdot}866 \times 10^{-7}$
1 inch ,, ,, =	$3{\cdot}388 \times 10^4$	$2{\cdot}95 \times 10^{-5}$
30 ,, ,, ,, =	$1{\cdot}0163 \times 10^6$	$9{\cdot}84 \times 10^{-7}$

Surface Tension (*in absolute measure*).

	Dynes per cm.	Reciprocals.
1 gm. per linear cm., - - =	981	·00102
1 grain ,, inch, - - =	25	·04
1 lb. ,, foot, - - =	14600	$6{\cdot}85 \times 10^{-5}$

Work and Energy (*in absolute measure*).

	Ergs.	Reciprocals.
1 gm. cm., - - =	981	·001019
1 kilogrammetre, =	$9{\cdot}81 \times 10^7$	$1{\cdot}019 \times 10^{-8}$
1 foot-pound, - =	$1{\cdot}356 \times 10^7$	$7{\cdot}37 \times 10^{-8}$
1 foot-poundal, -	= 421390	$2{\cdot}3731 \times 10^{-6}$

(The ratio of the ft.-poundal to the erg is independent of g.)

1 joule - - = 10^7 ergs.

Rate of Working (*in absolute measure*).

	Ergs per sec.	Reciprocals.
1 horse-power, - -	$= 7{\cdot}46 \times 10^9$	$1{\cdot}34 \times 10^{-10}$
1 force-de-cheval, - -	$= 7{\cdot}36 \times 10^9$	$1{\cdot}36 \times 10^{-10}$
1 watt, - - - -	= 10^7	10^{-7}

Heat (*in absolute measure*).

	Ergs.	Reciprocals.
1 gm. deg., - - -	$= 4{\cdot}2 \times 10^7$	$2{\cdot}38 \times 10^{-8}$
1 lb. deg. Cent., - -	$= 1{\cdot}905 \times 10^{10}$	$5{\cdot}25 \times 10^{-11}$
1 ,, Fahr., - -	$= 1{\cdot}058 \times 10^{10}$	$9{\cdot}45 \times 10^{-11}$

CHAPTER I.

GENERAL THEORY OF UNITS.

Units and Derived Units.

1. THE *numerical value* of a concrete quantity is its ratio to a selected magnitude of the same kind, called *the unit.*

Thus, if L denote a definite length, and l the unit length, $\frac{L}{l}$ is a ratio in the strict Euclidian sense, and is called the numerical value of L.

The numerical value of a concrete quantity varies directly as the concrete quantity itself, and inversely as the unit in terms of which it is expressed.

2. A unit of one kind of quantity is sometimes defined by reference to a unit of another kind of quantity. For example, the unit of *area* is commonly defined to be the area of the square described upon the unit of length; and the unit of *volume* is commonly defined as the volume of the cube constructed on the unit of length. The units of area and volume thus defined are called *derived units*, and are more convenient for calculation than independent units would be. For example, when the above

definition of the unit of area is employed, we can assert that [the numerical value of] the area of any rectangle is equal to the product of [the numerical values of] its length and breadth; whereas, if any other unit of area were employed, we should have to introduce a third factor which would be constant for all rectangles.

3. Still more frequently, a unit of one kind of quantity is defined by reference to two or more units of other kinds. For example, the unit of *velocity* is commonly defined to be that velocity with which the unit length would be described in the unit time. When we specify a velocity as so many *miles per hour*, or so many *feet per second*, we in effect employ as the unit of velocity a *mile per hour* in the former case, and a *foot per second* in the latter. These are *derived units* of velocity.

Again, the unit *acceleration* is commonly defined to be that acceleration with which a unit of velocity would be gained in a unit of time. The unit of acceleration is thus derived directly from the units of velocity and time, and therefore indirectly from the units of length and time.

4. In these and all other cases, the practical advantage of employing derived units is, that we thus avoid the introduction of additional factors, which would involve needless labour in calculating and difficulty in remembering.*

5. The correlative term to *derived* is *fundamental*. Thus, when the units of area, volume, velocity, and

* An example of such needless factors may be found in the rules commonly given in English books for finding the mass of a body when its volume and material are given. "Multiply the volume in cubic feet by the specific gravity and by 62·4, and the product will be the mass in pounds;" or "multiply the volume in cubic

acceleration are defined as above, the units of length and time are called the fundamental units.

Dimensions.

6. Let us now examine the laws according to which derived units vary when the fundamental units are changed.

Let V denote a concrete velocity such that a concrete length L is described in a concrete time T; and let v, l, t denote respectively the unit velocity, the unit length, and the unit time.

The numerical value of V is to be equal to the numerical value of L divided by the numerical value of T. But these numerical values are $\frac{V}{v}$, $\frac{L}{l}$, $\frac{T}{t}$;

hence we must have

$$\frac{V}{v} = \frac{L}{l}\,\frac{t}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This equation shows that, when the units are changed (a change which does not affect V, L, and T), v must vary directly as l and inversely as t; that is to say, *the unit of velocity varies directly as the unit of length, and inversely as the unit of time.*

Equation (1) also shows that *the numerical value* $\frac{V}{v}$ *of a given velocity varies inversely as the unit of length, and directly as the unit of time.*

inches by the specific gravity and by 253, and the product will be the mass in grains." The factors 62·4 and 253 here employed would be avoided—that is, would be replaced by unity, if the unit volume of water were made the unit of mass.

7. Again, let A denote a concrete acceleration such that the velocity V is gained in the time T′, and let a denote the unit of acceleration. Then, since the numerical value of the acceleration A is the numerical value of the velocity V divided by the numerical value of the time T′, we have

$$\frac{A}{a} = \frac{V}{v}\frac{t}{T'}.$$

But by equation (1) we may write $\frac{L}{l}\frac{t}{T}$ for $\frac{V}{v}$. We thus obtain

$$\frac{A}{a} = \frac{L}{l}\frac{t}{T}\frac{t}{T'}. \quad \ldots\ldots\ldots \quad (2)$$

This equation shows that when the units a, l, t are changed (a change which will not affect A, L, T or T′), a must vary directly as l, and inversely in the duplicate ratio of t; and the numerical value $\frac{A}{a}$ will vary inversely as l, and directly in the duplicate ratio of t. In other words, *the unit of acceleration varies directly as the unit of length, and inversely as the square of the unit of time;* and *the numerical value of a given acceleration varies inversely as the unit of length, and directly as the square of the unit of time.*

It will be observed that these have been deduced as direct consequences from the fact that [the numerical value of] an acceleration is equal to [the numerical value of] a length, divided by [the numerical value of] a time, and then again by [the numerical value of] a time.

The relations here pointed out are usually expressed by

saying that *the dimensions of acceleration** are $\frac{\text{length}}{(\text{time})^2}$, or that *the dimensions of the unit of acceleration** are

$$\frac{\text{unit of length}}{(\text{unit of time})^2}.$$

8. We have treated these two cases very fully, by way of laying a firm foundation for much that is to follow. We shall hereafter use an abridged form of reasoning, such as the following:—

$$\text{velocity} = \frac{\text{length}}{\text{time}};$$

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}} = \frac{\text{length}}{(\text{time})^2}.$$

Such equations as these may be called *dimensional equations*. Their full interpretation is obvious from what precedes. In all such equations, constant numerical factors can be discarded, as not affecting dimensions.

9. As an example of the application of equation (2) we shall compare the unit acceleration based on the foot and second with the unit acceleration based on the yard and minute.

Let l denote a foot, L a yard, t a second, T a minute, T′ a minute. Then a will denote the unit acceleration based on the foot and second, and A will denote the unit

* Professor James Thomson ('Brit. Assoc. Report,' 1878, p. 452) objects to these expressions, and proposes to substitute the following:—

"Change-ratio of unit of acceleration $= \frac{\text{change-ratio of unit of length}}{(\text{change-ratio of unit of time})^2}$"

This is very clear and satisfactory as a full statement of the meaning intended; but it is necessary to tolerate some abridgment of it for practical working.

acceleration based on the yard and minute. Equation (2) becomes

$$\frac{A}{a} = \frac{3}{1} \times \left(\frac{1}{60}\right)^2 = \frac{1}{1200}; \quad . \quad . \quad . \quad . \quad . \quad (3)$$

that is to say, an acceleration in which a yard per minute of velocity is gained per minute, is $\frac{1}{1200}$ of an acceleration in which a foot per second is gained per second.

Meaning of "per."

10. The word *per*, which we have several times employed in the present chapter, denotes division of the quantity named before it by the quantity named after it. Thus, to compute velocity in feet per second, we must divide a number of feet by a number of seconds.*

If velocity is continuously varying, let x be the number of feet described since a given epoch, and t the number of seconds elapsed, then $\frac{dx}{dt}$ is what is meant by the number of feet *per* second. The word should never be employed in the specification of quantities, except when the quantity named before it varies directly as the quantity named after it, at least for small variations—as, in the above instance, the distance described is ultimately proportional to the time of describing it.

Extended Sense of the terms "Multiplication" and "Division."

11. In ordinary multiplication the multiplier is always

* It is not correct to speak of interest at the rate of *Five Pounds per cent.* It should be simply *Five per cent.* A rate of five pounds in every hundred pounds is not different from a rate of five shillings in every hundred shillings.

a mere numerical quantity, and the product is of the same nature as the multiplicand. Hence in ordinary division either the divisor is a mere numerical quantity and the quotient a quantity of the same nature as the dividend; or else the divisor is of the same nature as the dividend, and the quotient a mere numerical quantity.

But in discussing problems relating to units, it is convenient to extend the meanings of the terms "multiplication" and "division." A distance divided by a time will denote a velocity—the velocity with which the given distance would be described in the given time. The distance can be expressed as a unit distance multiplied by a numerical quantity, and varies jointly as these two factors; the time can be expressed as a unit time multiplied by a numerical quantity, and is jointly proportional to *these* two factors. Also, the velocity remains unchanged when the time and distance are both changed in the same ratio.

12. The three quotients

$$\frac{1 \text{ mile}}{1 \text{ hour}}, \quad \frac{5280 \text{ ft.}}{3600 \text{ sec.}}, \quad \frac{22 \text{ ft.}}{15 \text{ sec.}}$$

all denote the same velocity, and are therefore to be regarded as equal. In passing from the first to the second, we have changed the units in the inverse ratio to their numerical multipliers, and have thus left both the distance and the time unchanged. In passing from the second to the third, we have divided the two numerical factors by a common measure, and have thus changed the distance and the time in the same ratio. A change in either factor of the numerator will be compensated by a proportional change in either factor of the denominator.

Further, since the velocity $\frac{22 \text{ ft.}}{15 \text{ sec.}}$ is $\frac{22}{15}$ of the velocity $\frac{1 \text{ ft.}}{1 \text{ sec.}}$, we are entitled to write $\frac{22 \text{ ft.}}{15 \text{ sec.}} = \frac{22}{15} \cdot \frac{\text{ft.}}{\text{sec.}}$, thus separating the numerical part of the expression from the units part.

In like manner we may express the result of Art. 9 by writing

$$\frac{\text{yard}}{(\text{minute})^2} = \frac{1}{1200} \cdot \frac{\text{foot}}{(\text{second})^2}$$

Such equations as these may be called "physical equations," inasmuch as they express the equality of physical quantities, whereas ordinary equations express the equality of mere numerical values. The use of physical equations in problems relating to units is to be strongly recommended, as affording a natural and easy clue to the necessary calculations, and especially as obviating the doubt by which the student is often embarrassed as to whether he ought to multiply or divide.

13. In the following examples, which illustrate the use of physical equations, we shall employ l to denote the unit length, m the unit mass, and t the unit time.

Ex. 1. If the yard be the unit of length, and the acceleration of gravity (in which a velocity of 32·2 ft. per sec. is gained per sec.) be represented by 2415, find the unit of time.

We have l = yard, and

$$32{\cdot}2 \frac{\text{ft.}}{(\text{sec.})^2} = 2415 \frac{l}{t^2} = 2415 \frac{3 \text{ ft.}}{t^2}$$

$$\therefore \ t^2 = \frac{2415 \times 3}{32{\cdot}2} \text{ sec.}^2 = 225 \text{ sec.}^2, \ t = 15 \text{ sec.}$$

Ex. 2. If the unit time be the second, the unit density 162 lbs. per cub. ft., and the unit force* the weight of an ounce at a place where g (in foot-second units) is 32, what is the unit length?

We have $t = \text{sec.}$, $\frac{m}{l^3} = 162 \frac{\text{lb.}}{(\text{ft.})^3}$,

and $\frac{ml}{\text{sec.}^2} = 32 \cdot \frac{\text{oz. ft.}}{\text{sec.}^2}$, or $ml = 32$ oz. ft. $= 2$ lb. ft.

Hence by division

$$l^4 = \frac{1}{81} (\text{ft.})^4, \; l = \tfrac{1}{3} \text{ ft.} = 4 \text{ in.}$$

Ex. 3. If the area of a field of 10 acres be represented by 100, and the acceleration of gravity (taken as 32 foot-second units) be $58\frac{2}{3}$, find the unit of time.

We have $48400 \; (\text{yd.})^2 = 100 \; l^2$, whence $l = 22$ yd.;

and $$32 \frac{\text{ft.}}{(\text{sec.})^2} = 58\tfrac{2}{3} \frac{l}{t^2} = \frac{176}{3} \frac{66 \text{ ft.}}{t^2},$$

whence $t^2 = \frac{176 \times 22}{32} \text{ sec.}^2 = 121 \text{ sec.}^2$, $t = 11$ sec.

Ex. 4. If 8 ft. per sec. be the unit velocity, and the acceleration of gravity (32 foot-second units) the unit acceleration, find the units of length and time.

We have the two equations

$$\frac{l}{t} = 8 \frac{\text{ft.}}{\text{sec.}}, \quad \frac{l}{t^2} = 32 \frac{\text{ft.}}{\text{sec.}^2},$$

whence by division $t = \frac{1}{4}$ sec., and substituting this value of t in the first equation, we have $4 \; l = 8$ ft., $l = 2$ ft.

Ex. 5. If the unit force be 100 lbs. weight, the unit length 2 ft., and the unit time $\frac{1}{4}$ sec., find the unit mass, the acceleration of gravity being taken as 32 foot-second units.

* For the dimensions of density and force, see Art. 14.

We have $l = 2$ ft., $t = \frac{1}{4}$ sec.,

$$100 \text{ lb. } 32 \frac{\text{ft.}}{\text{sec.}^2} = \frac{ml}{t^2} = \frac{m \ 2 \text{ ft.}}{\frac{1}{16} \text{sec.}^2},$$

that is 100×32 lb. $= 32\, m$, $m = 100$ lb.

Ex. 6. The number of seconds in the unit of time is equal to the number of feet in the unit of length, the unit of force is 750 lbs. weight [g being 32], and a cubic foot of the standard substance [substance of unit density] contains 13500 oz. Find the unit of time.

Let $t = x$ sec., then $l = x$ ft.; also let $m = y$ lb. Then we have

$$\frac{ml}{t^2} = \frac{y \text{ lb. } x \text{ ft.}}{x^2 \text{ sec.}^2} = \frac{y}{x} \frac{\text{lb. ft.}}{\text{sec.}^2} = 750 \times 32 \frac{\text{lb. ft.}}{\text{sec.}^2}$$

or
$$\frac{y}{x} = 750 \times 32.$$

Also
$$\frac{m}{l^3} = \frac{y}{x^3} \frac{\text{lb.}}{\text{ft.}^3} = 13500 \frac{\text{oz.}}{\text{ft.}^3};$$

whence
$$\frac{y}{x^3} = 13500 \times \frac{1}{16}.$$

Hence by division

$$x^2 = \frac{750 \times 32 \times 16}{13500} = \frac{16^2}{3^2}, \ x = \frac{16}{3}, \ t = \frac{16}{3} \text{sec.}$$

Ex. 7. When an inch is the unit of length and t the unit of time, the measure of a certain acceleration is a; when 5 ft. and 1 min. are the units of length and time respectively, the measure of the same acceleration is $10\, a$. Find t.

Equating the two expressions for the acceleration, we have
$$a \frac{\text{inch}}{t^2} = 10\, a \frac{5 \text{ ft.}}{(\text{min.})^2},$$

whence $t^2 = (\text{min.})^2 \frac{\text{inch}}{50 \text{ ft.}} = \frac{(\text{min.})^2}{600} = 6 (\text{sec.})^2$

$$t = \sqrt{6} \text{ sec.}$$

Ex. 8. The numerical value of a certain force is 56 when the pound is the unit of mass, the foot the unit of length, and the second the unit of time; what will be the numerical value of the same force when the hundredweight is the unit of mass, the yard the unit of length, and the minute the unit of time?

Denoting the required value by x we have

$$56 \frac{\text{lb. ft.}}{\text{sec.}^2} = x \frac{\text{cwt. yard}}{\text{min.}^2};$$

$$x = 56 \frac{\text{lb.}}{\text{cwt.}} \frac{\text{ft.}}{\text{yd.}} \left(\frac{\text{min.}}{\text{sec.}}\right)^2$$

$$= 56 \times \frac{1}{112} \times \frac{1}{3} \times 60^2 = 600.$$

Dimensions of Mechanical and Geometrical Quantities.

14. In the following list of dimensions, we employ the letters L, M, T as abbreviations for the words *Length*, *Mass*, *Time*. The symbol of equality is used to denote sameness of dimensions.

$$\text{Area} = \text{L}^2, \quad \text{Volume} = \text{L}^3, \quad \text{Velocity} = \frac{\text{L}}{\text{T}},$$

$$\text{Acceleration} = \frac{\text{L}}{\text{T}^2}, \quad \text{Momentum} = \frac{\text{ML}}{\text{T}}.$$

$\text{Density} = \frac{\text{M}}{\text{L}^3}$, density being defined as mass per unit volume.

$\text{Force} = \frac{\text{ML}}{\text{T}^2}$, since a force is measured by the momentum which it generates per unit of time, and is therefore

the quotient of momentum by time—or since a force is measured by the product of a mass by the acceleration generated in this mass.

Work $= \frac{ML^2}{T^2}$, being the product of force and distance.

Kinetic energy $= \frac{ML^2}{T^2}$, being half the product of mass by square of velocity. The constant factor $\frac{1}{2}$ can be omitted, as not affecting dimensions.

Moment of couple $= \frac{ML^2}{T^2}$, being the product of a force by a length.

The dimensions of *angle*,* when measured by $\frac{\text{arc}}{\text{radius}}$, are zero. The same angle will be denoted by the same number, whatever be the unit of length employed. In fact we have $\frac{\text{arc}}{\text{radius}} = \frac{L}{L} = L^0$.

The work done by a couple in turning a body through any angle, is the product of the couple by the angle. The identity of dimensions between *work* and *couple* is thus verified.

$$\text{Angular velocity} = \frac{1}{T}.$$

$$\text{Angular acceleration} = \frac{1}{T^2}.$$

$$\text{Moment of inertia} = ML^2.$$

Angular momentum = moment of momentum $= \frac{ML^2}{T}$,

* The name *radian* has been given to the angle whose arc is equal to radius. "An angle whose value in circular measure is θ" is "an angle of θ radians."

being the product of moment of inertia by angular velocity, or the product of momentum by length.

Intensity of pressure, or *intensity of stress* generally, being force per unit of area, is of dimensions $\frac{\text{force}}{\text{area}}$; that is, $\frac{M}{LT^2}$.

Intensity of force of *attraction at a point*, often called simply *force at a point*, being force per unit of attracted mass, is of dimensions $\frac{\text{force}}{\text{mass}}$ or $\frac{L}{T^2}$. It is numerically equal to the acceleration which it generates, and has accordingly the dimensions of acceleration.

The *absolute force of a centre* of attraction, better called the *strength of a centre*, may be defined as the intensity of force at unit distance. If the law of attraction be that of inverse squares, the strength will be the product of the intensity of force at any distance by the square of this distance, and its dimensions will be $\frac{L^3}{T^2}$.

Curvature (of a curve) $= \frac{1}{L}$, being the angle turned by the tangent per unit distance travelled along the curve.

Tortuosity $= \frac{1}{L}$, being the angle turned by the osculating plane per unit distance travelled along the curve.

The *solid angle* or *aperture* of a conical surface of any form is measured by the area cut off by the cone from a sphere whose centre is at the vertex of the cone, divided by the square of the radius of the sphere. Its dimensions are therefore zero; or a solid angle is a numerical quantity independent of the fundamental units.

The *specific curvature* of a surface at a given point (Gauss's measure of curvature) is the solid angle described by a line drawn from a fixed point parallel to the normal at a point which travels on the surface round the given point, and close to it, divided by the very small area thus enclosed. Its dimensions are therefore $\frac{1}{L^2}$.

The *mean curvature* of a surface at a given point, in the theory of Capillarity, is the arithmetical mean of the curvatures of any two normal sections normal to each other. Its dimensions are therefore $\frac{1}{L}$.

CHAPTER II.

CHOICE OF THREE FUNDAMENTAL UNITS.

15. NEARLY all the quantities with which physical science deals can be expressed in terms of three fundamental units; and the quantities commonly selected to serve as the fundamental units are

a definite length,
a definite mass,
a definite interval of time.

This particular selection is a matter of convenience rather than of necessity; for any three independent units are theoretically sufficient. For example, we might employ as the fundamental units

a definite mass,
a definite amount of energy,
a definite density.

16. The following are the most important considerations which ought to guide the selection of fundamental units:—

(1) They should be quantities admitting of very accurate comparison with other quantities of the same kind.

(2) Such comparison should be possible at all times. Hence the standards must be permanent—that is, not liable to alter their magnitude with lapse of time.

(3) Such comparisons should be possible at all places. Hence the standards must not be of such a nature as to change their magnitude when carried from place to place.

(4) The comparison should be easy and direct.

Besides these experimental requirements, it is also desirable that the fundamental units be so chosen that the definition of the various derived units shall be easy, and their dimensions simple.

17. There is probably no kind of magnitude which so completely fulfils the four conditions above stated as a standard of *mass*, consisting of a piece of gold, platinum, or some other substance not liable to be affected by atmospheric influences. The comparison of such a standard with other bodies of approximately equal mass is effected by weighing, which is, of all the operations of the laboratory, the most exact. Very accurate copies of the standard can thus be secured; and these can be carried from place to place with little risk of injury.

The third of the requirements above specified forbids the selection of a *force* as one of the fundamental units. Forces at the same place can be very accurately measured by comparison with weights; but as gravity varies from place to place, the force of gravity upon a piece of metal, or other standard weight, changes its magnitude in travelling from one place to another. A spring-balance, it is true, gives a direct measure of

force; but its indications are too rough for purposes of accuracy.

18. *Length* is an element which can be very accurately measured and copied. But every measuring instrument is liable to change its length with temperature. It is therefore necessary, in defining a length by reference to a concrete material standard, such as a bar of metal, to state the temperature at which the standard is correct. The temperature now usually selected for this purpose is that of a mixture of ice and water (0° C.), observation having shown that the temperature of such a mixture is constant.

The length of the standard should not exceed the length of a convenient measuring-instrument; for, in comparing the standard with a copy, the shifting of the measuring-instrument used in the comparison introduces additional risk of error.

In *end-standards*, the standard length is that of the bar as a whole, and the ends are touched by the instrument every time that a comparison is made. This process is liable to wear away the ends and make the standard false. In *line-standards*, the standard length is the distance between two scratches, and the comparison is made by optical means. The scratches are usually at the bottom of holes sunk halfway through the bar.

19. *Time* is also an element which can be measured with extreme precision. The direct instruments of measurement are clocks and chronometers; but these are checked by astronomical observations, and especially by transits of stars. The time between two successive transits of a star is (very approximately) the time of the

earth's rotation on its axis; and it is upon the uniformity of this rotation that the preservation of our standards of time depends.

Necessity for a Common Scale.

20. The existence of quantitative correlations between the various forms of energy, imposes upon men of science the duty of bringing all kinds of physical quantity to one common scale of comparison. Several such measures (called *absolute* measures) have been published in recent years; and a comparison of them brings very prominently into notice the great diversity at present existing in the selection of particular units of length, mass, and time.

Sometimes the units employed have been the foot, the grain, and the second; sometimes the millimetre, milligramme, and second; sometimes the centimetre, gramme, and second; sometimes the centimetre, gramme, and minute; sometimes the metre, tonne, and second; sometimes the metre, gramme, and second; while sometimes a mixture of units has been employed; the area of a plate, for example, being expressed in square metres, and its thickness in millimetres.

A diversity of scales may be tolerable, though undesirable, in the specification of such simple matters as length, area, volume, and mass when occurring singly; for the reduction of these from one scale to another is generally understood. But when the quantities specified involve a reference to more than one of the fundamental units, and especially when their dimensions in terms of these units are not obvious, but require careful working out,

there is great increase of difficulty and of liability to mistake.

A general agreement as to the particular units of length, mass, and time which shall be employed—if not in all scientific work, at least in all work involving complicated references to units—is urgently needed; and almost any one of the selections above instanced would be better than the present option.

21. We shall adopt the recommendation of the Units Committee of the British Association (see Appendix), that all specifications be referred to the *Centimetre*, the *Gramme*, and the *Second*. The system of units derived from these as the fundamental units is called the *C.G.S. system;* and the units of the system are called the *C.G.S. units.*

The reason for selecting the centimetre and gramme, rather than the metre and gramme, is that, since a gramme of water has a volume of approximately 1 cubic centimetre, the former selection makes the density of water unity; whereas the latter selection would make it a million, and the density of a substance would be a million times its specific gravity, instead of being identical with its specific gravity as in the C.G.S. system.

Even those who may have a preference for some other units will nevertheless admit the advantage of having a variety of results, from various branches of physics, reduced from their original multiplicity and presented in one common scale.

22. The adoption of one common scale for all quantities involves the frequent use of very large and very

small numbers. Such numbers are most conveniently written by expressing them as the product of two factors, one of which is a power of 10; and it is usually advantageous to effect the resolution in such a way that the exponent of the power of 10 shall be the characteristic of the logarithm of the number. Thus 3240000000 will be written $3{\cdot}24 \times 10^{9}$, and ·00000324 will be written $3{\cdot}24 \times 10^{-6}$.

CHAPTER III.

MECHANICAL UNITS.

Value of g.

23. ACCELERATION is defined as the rate of increase of velocity per unit of time. The C.G.S. unit of acceleration is the acceleration of a body whose velocity increases in every second by the C.G.S. unit of velocity—namely, by a centimetre per second. The apparent acceleration of a body falling freely under the action of gravity in vacuo is denoted by g. The value of g in C.G.S. units at any part of the earth's surface is approximately given by the following formula,

$$g = 980{\cdot}6056 - 2{\cdot}5028 \cos 2\lambda - {\cdot}0000003h,$$

λ denoting the latitude, and h the height of the station (in centimetres) above sea-level.

The constants in this formula have been deduced from numerous pendulum experiments in different localities, the length l of the seconds' pendulum being connected with the value of g by the formula $g = \pi^2 l$.

Dividing the above equation by π^2 we have, for the length of the seconds' pendulum, in centimetres,

$$l = 99{\cdot}3562 - {\cdot}2536 \cos 2\lambda - {\cdot}0000003h.$$

At sea-level these formulæ give the following values for the places specified :—

	Latitude.	Value of g.	Value of l.
Equator, - - -	° ′ 0 0	978·10	99·103
Latitude 45°, - -	45 0	980·61	99·356
Munich, - - -	48 9	980·88	99·384
Paris, - - -	48 50	980·94	99·390
Greenwich, - -	51 29	981·17	99·413
Göttingen, - -	51 32	981·17	99·414
Berlin, - - -	52 30	981·25	99·422
Dublin, - - -	53 21	981·32	99·429
Manchester, - -	53 29	981·34	99·430
Belfast, - - -	54 36	981·43	99·440
Edinburgh, - -	55 57	981·54	99·451
Aberdeen, - -	57 9	981·64	99·461
Pole, - - -	90 0	983·11	99·610

The difference between the greatest and least values (in the case of both g and l) is about $\frac{1}{196}$ of the mean value.

24. The Standards Department of the Board of Trade, being concerned only with relative determinations, has adopted the formula

$$g = g_0(1 - \cdot00257 \cos 2\lambda)\left(1 - \frac{5}{4}\frac{h}{R}\right),$$

λ denoting the latitude, h the height above sea-level, R the earth's radius, g_0 the value of g in latitude 45° at sea-level, which may be treated as an unknown constant multiplier. Putting for R its value in centimetres, the formula gives

$$g = g_0(1 - \cdot00257 \cos 2\lambda - 1\cdot96h \times 10^{-9}),$$

where h denotes the height in centimetres.

The formula which we employed in the preceding section gives

$$g = g_0(1 - \cdot00255 \cos 2\lambda)\left(1 - \frac{2h}{R}\right).$$

As regards the factor dependent on height, theory indicates $1 - \frac{2h}{R}$ as its correct value for such a case as that of a balloon in mid-air over a low-lying country; the value $1 - \frac{5}{4}\frac{h}{R}$ may be accepted as more correct for an elevated plateau on the earth's surface.

Force.

25. The C.G.S. unit of force is called the *dyne*. It is the force which, acting upon a gramme for a second, generates a velocity of a centimetre per second.

It may otherwise be defined as the force which, acting upon a gramme, produces the C.G.S. unit of acceleration, or as the force which, acting upon any mass for 1 second, produces the C.G.S. unit of momentum.

To show the equivalence of these three definitions, let m denote mass in grammes, v velocity in centimetres per second, t time in seconds, F force in dynes.

Then, by the second law of motion, we have

$$\text{acceleration} = \frac{\text{force}}{\text{mass}};$$

that is, if a denote acceleration in C.G.S. units, $a = \frac{F}{m}$; hence, when a and m are each unity, F will be unity.

Again, by the nature of uniform acceleration, we have $v = at$, v denoting the velocity due to the acceleration a, continuing for time t.

Hence we have $F = ma = \frac{mv}{t}$. Therefore, if $mv = 1$ and $t = 1$, we have $F = 1$.

As a particular case, if $m = 1$, $v = 1$, $t = 1$, we have $F = 1$.

26. The force represented by the *weight of a gramme* varies from place to place. It is the force required to sustain a gramme in vacuo, and would be nil at the earth's centre, where gravity is nil. To compute its amount in dynes at any place where g is known, observe that a mass of 1 gramme falls in vacuo with acceleration g. The force producing this acceleration (namely, the weight of the gramme) must be equal to the product of the mass and acceleration, that is, to g.

The weight (when weight means force) of 1 gramme is therefore g dynes; and the weight of m grammes is mg dynes.

27. Force is said to be expressed in *gravitation-measure* when it is expressed as equal to the weight of a given mass. Such specification is inexact unless the value of g is also given. For purposes of accuracy it must always be remembered that the pound, the gramme, etc., are, strictly speaking, units of mass. Such an expression as "a force of 100 tons" must be understood as an abbreviation for "a force equal to the weight [at the locality in question] of 100 tons."

28. The name *poundal* has recently been given to the unit force based on the pound, foot, and second; that is, the force which, acting on a pound for a second, generates a velocity of a foot per second. It is $\frac{1}{g}$ of the

weight of a pound, g denoting the acceleration due to gravity expressed in foot-second units, which is about 32·2 in Great Britain.

To compare the poundal with the dyne, let x denote the number of dynes in a poundal; then we have

$$x \frac{\text{gm. cm.}}{\text{sec.}^2} = \frac{\text{lb. ft.}}{\text{sec.}^2}$$

$$x = \frac{\text{lb.}}{\text{gm.}} \cdot \frac{\text{ft.}}{\text{cm.}} = 453{\cdot}59 \times 30{\cdot}4797 = 13825.$$

Work and Energy.

29. The C.G.S. unit of work is called the *erg*. It is the amount of work done by a dyne working through a distance of a centimetre.

The C.G.S. unit of energy is also the erg, energy being measured by the amount of work which it represents.

30. To establish a rule for computing the *kinetic energy* (or *energy due to the motion*) of a given mass moving with a given velocity, it is sufficient to consider the case of a body falling in vacuo.

When a body of m grammes falls through a height of h centimetres, the working force is the weight of the body —that is, gm dynes, which, multiplied by the distance worked through, gives gmh ergs as the work done. But the velocity acquired is such that $v^2 = 2gh$. Hence we have $gmh = \frac{1}{2}mv^2$.

The kinetic energy of a mass of m grammes moving with a velocity of v centimetres per second is therefore $\frac{1}{2}mv^2$ ergs; that is to say, this is the amount of work which would be required to generate the motion of the body, or is the amount of work which the body

would do against opposing forces before it would come to rest.

31. Work, like force, is often expressed in *gravitation-measure*. Gravitation units of work, such as the foot-pound and kilogramme-metre, vary with locality, being proportional to the value of g.

One gramme-centimetre is equal to g ergs.

One kilogramme-metre is equal to 100,000 g ergs.

One foot-poundal is $453{\cdot}59 \times (30{\cdot}4797)^2 = 421390$ ergs.

One foot-pound is 13,825 g ergs, which, if g be taken as 981, is $1{\cdot}356 \times 10^7$ ergs.

32. The C.G.S. unit rate of working is 1 erg per second. Watt's "horse-power" is defined as 550 foot-pounds per second. This is $7{\cdot}46 \times 10^9$ ergs per second. The "force de cheval" is defined as 75 kilogrammetres per second. This is $7{\cdot}36 \times 10^9$ ergs per second. We here assume $g = 981$.

A new unit of rate of working has been lately introduced for convenience in certain electrical calculations. It is called the *Watt*, and is defined as 10^7 ergs per second. A thousand watts make a *kilowatt*. The following tabular statement will be useful for reference.

1 Watt = 10^7 ergs per second = ·00134 horse-power = ·737 foot-pounds per second = ·1019 kilogrammetres per second.

1 Kilowatt = 1·34 horse-power.

1 Horse-power = 550 foot-pounds per second = 76·0 kilogrammetres per second = 746 watts = 1·01385 force de cheval.

1 Force de cheval = 75 kilogrammetres per second = 542·48 foot-pounds per second = 736 watts = ·9863 horse-power.

Examples.

1. If a spring balance is graduated so as to show the masses of bodies in pounds or grammes when used at the equator, what will be its error when used at the poles, neglecting effects of temperature?

Ans. Its indications will be too high by about $\frac{1}{196}$ of the total weight.

2. A cannon-ball, of 10,000 grammes, is discharged with a velocity of 45,000 centims. per second. Find its kinetic energy.

Ans. $\frac{1}{2} \times 10000 \times (45000)^2 = 1{\cdot}0125 \times 10^{13}$ ergs.

3. In last question find the mean force exerted upon the ball by the powder, the length of the barrel being 200 centims.

Ans. $5{\cdot}0625 \times 10^{10}$ dynes.

4. Given that 42 million ergs are equivalent to 1 gramme-degree of heat, and that a gramme of lead at 10° C. requires 15·6 gramme-degrees of heat to melt it; find the velocity with which a leaden bullet must strike a target that it may just be melted by the collision, supposing all the mechanical energy of the motion to be converted into heat and to be taken up by the bullet.

We have $\frac{1}{2}v^2 = 15{\cdot}6 \times J$, where $J = 42 \times 10^6$. Hence $v^2 = 1310$ millions; $v = 36{\cdot}2$ thousand centims. per second.

5. With what velocity must a stone be thrown vertically upwards at a place where g is 981 that it may rise to a height of 3000 centims.? and to what height would it ascend if projected vertically with this velocity at the surface of the moon, where g is 150?

Ans. 2426 centims. per second; 19620 centims.

Centrifugal Force.

33. A body moving in a curve must be regarded as continually falling away from a tangent. The acceleration with which it falls away is $\frac{v^2}{r}$, v denoting its velocity and r the radius of curvature. The acceleration of a body in any direction is always due to force urging it in that direction, this force being equal to the product of mass and acceleration. Hence the normal force on a body of m grammes moving in a curve of radius r centimetres, with velocity v centimetres per second, is $\frac{mv^2}{r}$ dynes. This force is directed towards the centre of curvature. The equal and opposite force with which the body reacts is called centrifugal force.

If the body moves uniformly in a circle, the time of revolution being T seconds, we have $v = \frac{2\pi r}{T}$; hence $\frac{v^2}{r} = \left(\frac{2\pi}{T}\right)^2 r$, and the force acting on the body is $mr\left(\frac{2\pi}{T}\right)^2$ dynes.

If n revolutions are made per minute, the value of T is $\frac{60}{n}$, and the force is $mr\left(\frac{n\pi}{30}\right)^2$ dynes.

Examples.

1. A body of m grammes moves uniformly in a circle of radius 80 centims., the time of revolution being $\frac{1}{4}$ of a

second. Find the centrifugal force, and compare it with the weight of the body.

Ans. The centrifugal force is $m \times \left(\frac{2\pi}{\frac{1}{4}}\right)^2 \times 80 = m \times 64\pi^2 \times 80 = 50532\ m$ dynes.

The weight of the body (at a place where g is 981) is 981 m dynes. Hence the centrifugal force is about $52\frac{1}{2}$ times the weight of the body.

2. At a bend in a river, the velocity in a certain part of the surface is 170 centims. per second, and the radius of curvature of the lines of flow is 9100 centims. Find the slope of the surface in a section transverse to the lines of flow.

Ans. Here the centrifugal force for a gramme of the water is $\frac{(170)^2}{9100} = 3{\cdot}176$ dynes. If g be 981 the slope will be $\frac{3{\cdot}176}{981} = \frac{1}{309}$; that is, the surface will slope upwards from the concave side at a gradient of 1 in 309. The general rule applicable to questions of this kind is that the resultant of centrifugal force and gravity must be normal to the surface.

3. An open vessel of liquid is made to rotate rapidly round a vertical axis. Find the number of revolutions that must be made per minute in order to obtain a slope of 30° at a part of the surface distant 10 centims. from the axis, the value of g being 981.

Ans. We must have $\tan 30° = \frac{f}{g}$, where f denotes the intensity of centrifugal force—that is, the centrifugal force per unit mass. We have therefore

$$981 \tan 30° = 10\left(\frac{n\pi}{30}\right)^2, \quad n \text{ denoting the number of revolutions per minute,}$$

$$= \frac{n^2\pi^2}{90}.$$

Hence $n = 71{\cdot}9$.

4. For the intensity of centrifugal force at the equator due to the earth's rotation, we have r = earth's radius $= 6{\cdot}38 \times 10^8$, $T = 86164$, being the number of seconds in a sidereal day.

$$\therefore \quad f = r\left(\frac{2\pi}{T}\right)^2 = 3{\cdot}39.$$

This is about $\frac{1}{289}$ of the value of g.

If the earth were at rest, the value of g at the equator would be greater than at present by this amount. If the earth were revolving about 17 times as fast as at present, the value of g at the equator would be nil.

Supplemental Section.

On the help to be derived from Dimensions in investigating Physical Formulæ.

When one physical quantity is known to vary as some power of another physical quantity, it is often possible to find the exponent of this power by reasoning based on dimensions, and thus to anticipate the results—or some of the results—of a dynamical investigation.

Examples.

1. The time of vibration of a simple pendulum in a small arc depends on the length of the pendulum and the intensity of gravity. If we assume it to vary as the m^{th}

power of the length, and as the n^{th} power of g, and to be independent of everything else, the dimensions of a time must equal the m^{th} power of a length, multiplied by the n^{th} power of an acceleration, that is

$$T = L^m (LT^{-2})^n = L^m L^n T^{-2n}$$
$$= L^{m+n} T^{-2n}.$$

Since the dimensions of both members are to be identical, we have, by equating the exponents of T,

$$1 = -2n, \text{ whence } n = -\tfrac{1}{2},$$

and by equating the exponents of L,

$$m + n = 0, \text{ whence } m = \tfrac{1}{2};$$

that is, the time of vibration varies directly as the square root of the length, and inversely as the square root of g.

2. The velocity of sound in a gas depends only on the density D of the gas and its coefficient of elasticity E, and we shall assume it to vary as $D^m E^n$.

The dimensions of velocity are LT^{-1}.

The dimensions of density, or $\frac{\text{mass}}{\text{volume}}$, are ML^{-3}.

The dimensions of E, which will be explained in the chapter on stress and strain, are $\frac{\text{force}}{\text{area}}$, or $(MLT^{-2})L^{-2}$, or $ML^{-1}T^{-2}$.

The equation of dimensions is

$$LT^{-1} = M^m L^{-3m} \cdot M^n L^{-n} T^{-2n},$$
$$= M^{m+n} L^{-3m-n} T^{-2n},$$

whence, by equating coefficients, we have the three equations

$$1 = -3m - n, \quad -1 = -2n, \quad m + n = 0,$$

to determine the two unknowns m and n.

The second equation gives at once

$$n = \tfrac{1}{2}.$$

The third then gives

$$m = -\tfrac{1}{2},$$

and these values will be found to satisfy the first equation also.

The velocity, then, varies directly as the square root of E, and inversely as the square root of D.

3. The frequency of vibration f for a musical string (that is, the number of vibrations per unit time) depends on its length l, its mass m, and the force with which it is stretched F.

The dimensions of f are T^{-1}.

,, ,, F ,, MLT^{-2}.

Assume that f varies as $l^x m^y F^z$. Then we have

$$T^{-1} = L^x M^y M^z L^z T^{-2z},$$
$$= L^{x+z} M^{y+z} T^{-2z},$$

giving $-1 = -2z, \quad x+z=0, \quad y+z=0;$

whence $z = \tfrac{1}{2}, \quad x = -\tfrac{1}{2}, \quad y = -\tfrac{1}{2}.$

Hence f varies as $\sqrt{\dfrac{F}{lm}}$.

4. The angular acceleration of a uniform disc round its axis depends on the applied couple G, the mass of the disc M, and its radius R.

Assume it to vary as $G^x M^y R^z$.

The dimensions of angular acceleration are T^{-2}.

,, ,, G ,, ML^2T^{-2}.

,, ,, R ,, L.

Hence we have

$$T^{-2} = M^x L^{2x} T^{-2x} M^y L^z,$$
$$= M^{x+y} L^{2x+z} T^{-2x};$$

giving $-2 = -2x, \quad x+y=0, \quad 2x+z=0,$

whence $x=1, \quad y=-1, \quad z=-2.$

Hence the angular acceleration varies as $\frac{G}{MR^2}$.

In the following example the information obtained is less complete :—

5. The range of a projectile on a horizontal plane through the point of projection depends on the initial velocity V, the intensity of gravity g, and the angle of elevation α.

The dimensions of range are L.

" " V " LT^{-1}.

" " g " LT^{-2}.

" " α " L^0T^0, and the dimensions of all powers of α are L^0T^0. Hence we can draw no inferences as to the manner in which α enters the expression for the range. The dimensions of this expression will depend upon V and g alone.

Assume that the range varies as $V^m g^n$. Then

$$L = (LT^{-1})^m (LT^{-2})^n,$$

$$= L^{m+n} T^{-m-2n};$$

giving $m+n=1, \quad m+2n=0,$

whence $m=2, \quad n=-1.$

Hence the range varies as $\frac{V^2}{g}$ when α is given.

CHAPTER IV.

HYDROSTATICS.

34. The following table of the relative density of water at various temperatures (under atmospheric pressure), the density at 4° C. being taken as unity, is from Rossetti's results deduced from all the best experiments (Ann. Ch. Phys. x. 461; xvii. 370, 1869):—

Temp. Cent.	Relative Density.	Temp. Cent.	Relative Density.	Temp. Cent.	Relative Density.
0°	·999871	13°	·999430	35°	·99418
1	·999928	14	·999299	40	·99235
2	·999969	15	·999160	45	·99037
3	·999991	16	·999002	50	·98820
4	1·000000	17	·998841	55	·98582
5	·999990	18	·998654	60	·98338
6	·999970	19	·998460	65	·98074
7	·999933	20	·998259	70	·97794
8	·999886	22	·997826	75	·97498
9	·999824	24	·997367	80	·97194
10	·999747	26	·996866	85	·96879
11	·999655	28	·996331	90	·96556
12	·999549	30	·995765	100	·95865

35. According to Kupffer's observations, as reduced by Professor W. H. Miller, the absolute density (in grammes per cubic centimetre) at 4° is not 1, but 1·000013. Multiplying the above numbers by this

factor, we obtain the following table of absolute densities :—

Temp	Density.	Temp.	Density.	Temp.	Density.
0°	·999884	13°	·999443	35°	·99469
1	·999941	14	·999312	40	·99236
2	·999982	15	·999173	45	·99038
3	1·000004	16	·999015	50	·98821
4	1·000013	17	·998854	55	·98583
5	1·000003	18	·998667	60	·98339
6	·999983	19	·998473	65	·98075
7	·999946	20	·998272	70	·97795
8	·999899	22	·997839	75	·97499
9	·999837	24	·997380	80	·97195
10	·999760	26	·996879	85	·96880
11	·999668	28	·996344	90	·96557
12	·999562	30	·995778	100	·95866

36. The volume, at temperature t', of the water which occupies unit volume at 4°, is approximately

$$1 + A(t-4)^2 - B(t-4)^{2\cdot6} + C(t-4)^3,$$

where

$$A = 8\cdot38 \times 10^{-6},$$
$$B = 3\cdot79 \times 10^{-7},$$
$$C = 2\cdot24 \times 10^{-8};$$

and the relative density at temperature $t°$ is given by the same formula with the signs of A, B, and C reversed. The rate of expansion at temperature $t°$ is

$$2A(t-4) - 2\cdot6B(t-4)^{1\cdot6} \times 3C(t-4)^2.$$

In determining the signs of the terms with the fractional exponents 2·6 and 1·6, these exponents are to be regarded as odd.

37. The following Table of Densities has been compiled by collating the best authorities, but is only to be taken

as giving rough approximations. Most of the densities vary between wide limits in different specimens:—

Solids.

Aluminium,	2·6	Carbon (diamond),	3·5
Antimony,	6·7	,, (graphite),	2·3
Bismuth,	9·8	,, (gas carbon),	1·9
Brass,	8·4	,, (wood charcoal),	1·6
Copper,	8·9	Phosphorus (ordinary),	1·83
Gold,	19·3	,, (red),	2·2
Iron,	7·8	Sulphur (roll),	2·0
Lead,	11·3	Quartz (rock crystal),	2·65
Nickel,	8·9	Sand (dry),	1·42
Platinum,	21·5	Clay,	1·9
Silver,	10·5	Brick,	2·1
Sodium,	·98	Basalt,	3·0
Tin,	7·3	Chalk,	1·8 to 2·8
Zinc,	7·1	Glass (crown),	2·5 to 2·7
Cork,	·24	,, (flint),	3·0 to 3·5
Oak,	·7 to 1·0	Porcelain,	2·4
Ebony,	1·1 to 1·2		
Ice,	·918		

Liquids at 0° C.

Sea water,	1·026	Sulphuric Acid,	1·85
Alcohol,	·8	Nitric Acid,	1·56
Chloroform,	1·5	Hydrochloric Acid,	1·27
Ether,	·73	Milk,	1·03
Bisulphide of Carbon,	1·29	Oil of Turpentine,	·87
Glycerine,	1·27	,, Linseed,	·94
Mercury,	13·596	,, Mineral,	·76 to ·83

More exactly, the density of mercury at 0° C., as compared with water at the temperature of maximum density, under atmospheric pressure, is 13·5956.

38. If a body weighs m grammes in vacuo and m' grammes in water of density unity, the volume of the body is $m - m'$ cubic centims.; for the mass of the water displaced is $m - m'$ grammes, and each gramme of this water occupies a cubic centimetre.

Examples.

1. A glass cylinder, l centims. long, weighs m grammes in vacuo and m' grammes in water of unit density. Find its radius.

Solution. Its section is πr^2, and is also $\frac{m-m'}{l}$; hence

$$r^2 = \frac{m-m'}{\pi l}.$$

2. Find the capacity at 0° C. of a bulb which holds m grammes of mercury at that temperature.

Solution. The specific gravity of mercury at 0° being 13·5956 as compared with water at the temperature of maximum density, it follows that the mass of 1 cubic centim. of mercury is $13{\cdot}5956 \times 1{\cdot}000013 = 13{\cdot}5958$, say 13·596. Hence the required capacity is $\frac{m}{13{\cdot}596}$ cubic centims.

3. Find the total pressure on a surface whose area is A square centims. when its centre of gravity is immersed to a depth of h centims. in water of unity density, atmospheric pressure being neglected.

Ans. Ah grammes weight; that is, gAh dynes.

4. If mercury of specific gravity 13·596 is substituted for water in the preceding question, find the pressure.

Ans. 13·596 Ah grammes weight; that is, 13·596 gAh dynes.

5. If h be 76, and A be unity in example 4, the answer becomes 1033·3 grammes weight, or $1033{\cdot}3g$ dynes.

For Paris, where g is 980·94, this is $1{\cdot}0136 \times 10^6$ dynes.

Barometric Pressure.

39. The C.G.S. unit of pressure intensity (that is, of pressure per unit area) is the pressure of a dyne per square centim.

At the depth of h centims. in a uniform liquid whose density is d [grammes per cubic centim.], the pressure due to the weight of the liquid is ghd dynes per square centim.

The pressure-intensity due to the weight of a column of mercury at 0° C., 76 centims. high, is found by putting $h = 76$, $d = 13{\cdot}596$, and is $1033{\cdot}3g$. It is therefore different at different localities. At Paris, where g is 980·94, it is $1{\cdot}0136 \times 10^6$; that is, rather more than a megadyne* per square centim. To exert a pressure of exactly one megadyne per square centim., the height of the column at Paris must be 74·98 centims.

At Greenwich, where g is 981·17, the pressure due to 76 centims. of mercury at 0° C. is $1{\cdot}0138 \times 10^6$; and the height which would give a pressure of 10^6 is 74·964 centims., or 29·514 inches.

Convenience of calculation would be promoted by adopting the pressure of a megadyne per square centim., or 10^6 C.G.S. units of pressure-intensity, as the standard atmosphere.

The standard now commonly adopted (whether 76 centims. or 30 inches) denotes different pressures at different places, the pressure denoted by it being proportional to the value of g.

We shall adopt the megadyne per square centim. as our standard atmosphere in the present work.

*The prefix *mega* denotes multiplication by a million. A megadyne is a force of a million dynes.

Examples.

1. What must be the height of a column of water of unit density to exert a pressure of a megadyne per square centim. at a place where g is 981?

Ans. $\frac{1000000}{981} = 1019{\cdot}4$ centims. This is 33·445 feet.

2. What is the pressure due to an inch of mercury at 0° C. at a place where g is 981? (An inch is 2·54 centims.)

Ans. $981 \times 2{\cdot}54 \times 13{\cdot}596 = 33878$ dynes per square centim.

3. What is the pressure due to a centim. of mercury at 0° C. at the same locality?

Ans. $981 \times 13{\cdot}596 = 13338$.

4. What is the pressure due to a kilometre of sea-water of density 1·027, g being 981?

Ans. $981 \times 10^5 \times 1{\cdot}027 = 1{\cdot}0075 \times 10^8$ dynes per square centim., or $1{\cdot}0075 \times 10^2$ megadynes per square centim.; that is, about 100 atmospheres.

5. What is the pressure due to a mile of the same water?

Ans. $1{\cdot}6214 \times 10^8$ C.G.S. units, or 162·14 atmospheres [of a megadyne per square centim.].

Density of Air.

40. Regnault found that at Paris, under the pressure of a column of mercury at 0°, of the height of 76 centims., the density of perfectly dry air was ·0012932 gramme per cubic centim. The pressure corresponding to this height of the barometer at Paris is $1{\cdot}0136 \times 10^6$ dynes per square

centim. Hence, by Boyle's law, we can compute the density of dry air at 0° C. at any given pressure.

At a pressure of a megadyne (10^6 dynes) per square centim. the density will be $\frac{\cdot 0012932}{1\cdot 0136} = \cdot 0012759$.

The density of dry air at 0° C. at any pressure p (dynes per square centim.) is

$$p \times 1\cdot 2759 \times 10^{-9} \quad . \quad . \quad . \quad . \quad (4)$$

Example.

Find the density of dry air at 0° C., at Edinburgh, under the pressure of a column of mercury at 0° C., of the height of 76 centims.

Here we have $p = 981\cdot 54 \times 76 \times 13\cdot 596 = 1\cdot 0142 \times 10^6$.

Ans. Required density $= 1\cdot 2940 \times 10^{-3} = \cdot 0012940$ gramme per cubic centim.

41. *Absolute Densities of Gases, in grammes per cubic centim., at* 0° C., *and a pressure of* 10^6 *dynes per square centim.*

	Mass of a cubic centim. in grammes.	Volume of a gramme in cubic centims.
Air, dry,	·0012759	783·8
Oxygen,	·0014107	708·9
Nitrogen,	·0012393	806·9
Hydrogen,	·00008837	11316·0
Carbonic Acid,	·0019509	512·6
,, Oxide,	·0012179	821·1
Marsh Gas,	·0007173	1394·1
Chlorine,	·0030909	323·5
Protoxide of Nitrogen,	·0019433	514·6
Binoxide ,,	·0013254	754·5
Sulphurous Acid,	·0026990	370·5
Cyanogen,	·0022990	435·0
Olefiant Gas,	·0012529	798·1
Ammonia,	·0007594	1316·8

The numbers in the second column are the reciprocals of those in the first.

The numbers in the first column are identical with the specific gravities referred to water as unity.

Assuming that the densities of gases at given pressure and temperature are directly as their atomic weights, we have for any gas at zero

$$pv\mu = 1{\cdot}1316 \times 10^{10} m\ ;$$

v denoting its volume in cubic centims., m its mass in grammes, p its pressure in dynes per square centim., and μ its atomic weight referred to that of hydrogen as unity.

Height of Homogeneous Atmosphere.

42. We have seen that the intensity of pressure at depth h, in a fluid of uniform density d, is ghd when the pressure at the upper surface of the fluid is zero.

The atmosphere is not a fluid of uniform density; but it is often convenient to have a name to denote a height H such that $p = g\mathrm{HD}$, where p denotes the pressure and D the density of the air at a given point.

It may be defined as the height of a column of uniform fluid having the same density as the air at the point, which would exert a pressure equal to that existing at the point.

If the pressure be equal to that exerted by a column of mercury of density 13·596 and height h, we have

$$p = gh \times 13{\cdot}596\ ;$$

$$\therefore \quad \mathrm{HD} = h \times 13{\cdot}596, \quad \mathrm{H} = \frac{h \times 13{\cdot}596}{\mathrm{D}}.$$

If it were possible for the whole body of air above the point to be reduced by vertical compression to the density which the air has at the point, the height from the point

up to the summit of this compressed atmosphere would be equal to H, subject to a small correction for the variation of gravity with height.

H is called the *height of the homogeneous atmosphere* at the point considered. *Pressure-height* would be a better name.

The general formula for it is

$$H = \frac{p}{gD}; \quad . \quad . \quad . \quad . \quad (5)$$

and this formula will be applicable to any other gas as well as dry air, if we make D denote the density of the gas (in grammes per cubic centim.) at pressure p.

If, instead of p being given directly in dynes per square centim., we have given the height h of a column of liquid of density d which would exert an equal pressure, the formula reduces to

$$H = \frac{hd}{D}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

43. The value of $\frac{p}{D}$ in formula (5) depends only on the nature of the gas and on the temperature; hence, for a given gas at a given temperature, H varies inversely as g only.

For dry air at zero we have, by formula (4),

$$\frac{p}{D} = \frac{10^9}{1{\cdot}2759} = 7{\cdot}8376 \times 10^8;$$

$$\therefore \quad H = \frac{7{\cdot}8376 \times 10^8}{g}.$$

At Paris, where g is 980·94, we find

$$H = 7{\cdot}990 \times 10^5.$$

At Greenwich, where g is 981·17,

$$H = 7{\cdot}988 \times 10^5.$$

Examples.

1. Find the height of the homogeneous atmosphere at Paris for dry air at 10° C., and also at 100° C.

Ans. For given density, p varies as $1 \times \cdot00366\ t$, t denoting the temperature on the Centigrade scale. Hence we have, at 10° C.,

$$H = 1{\cdot}0366 \times 7{\cdot}99 \times 10^5 = 8{\cdot}2825 \times 10^5;$$

and at 100° C.,

$$H = 1{\cdot}366 \times 7{\cdot}99 \times 10^5 = 1{\cdot}0914 \times 10^6.$$

2. Find the height of the homogeneous atmosphere for hydrogen at 0°, at a place where g is 981.

Here we have

$$H = \frac{p}{gd} = \frac{10^6}{981 \times 8{\cdot}837 \times 10^{-5}} = 1{\cdot}1535 \times 10^7.$$

Diminution of Density with increase of Height in the Atmosphere.

44. Neglecting the variation of gravity with height, the variation of H as we ascend in the atmosphere would depend only on variation of temperature. In an atmosphere of uniform temperature H will be the same at all heights. In such an atmosphere, an ascent of 1 centim. will involve a diminution of the pressure (and therefore of the density) by $\frac{1}{H}$ of itself, since the layer of air which has been traversed is $\frac{1}{H}$ of the whole mass of superincumbent air. The density therefore diminishes by the same fraction of itself for every centim. that we ascend; in other words, the density and pressure diminish in geometrical progression as the height increases in arithmetical progression.

Denote height above a fixed level by x, and pressure by p. Then, in the notation of the differential calculus, we have

$$\frac{dx}{H} = -\frac{dp}{p},$$

and if p_1, p_2 are the pressures at the heights x_1, x_2, we deduce

$$x_2 - x_1 = H \log_e \frac{p_1}{p_2} = H \times 2{\cdot}3026 \log_{10} \frac{p_1}{p_2}. \quad . \quad . \quad (7)$$

In the barometric determination of heights it is usual to compute H by assuming a temperature which is the arithmetical mean of the temperatures at the two heights.

For the latitude of Greenwich formula (7) becomes

$$x_2 - x_1 = (1 \times {\cdot}00366\, t)\, 7{\cdot}988 \times 10^5 \times 2{\cdot}3026 \log \frac{p_1}{p_2}$$

$$= (1 \times {\cdot}00366\, t)\, 1{,}839{,}300 \log \frac{p_1}{p_2}, \quad . \quad . \quad (8)$$

t denoting the mean temperature, and the logarithms being common logarithms.

To find the height at which the density would be halved, variations of temperature being neglected, we must put 2 for $\frac{p_1}{p_2}$ in these formulæ. The required height will be H $\log_e 2$, or, in the latitude of Greenwich, for temperature 0° C., will be

$$1{\cdot}8393 \times 10^6 \times {\cdot}30103 = 553700.$$

The value of $\log_e 2$, or $2{\cdot}3026 \log_{10} 2$, is

$$2{\cdot}3026 \times {\cdot}30103 = {\cdot}69315.$$

Hence for an atmosphere of any gas at uniform temperature, the height at which the density would be halved is the height of the homogeneous atmosphere for that gas, multiplied by ·69315. The gas is assumed to obey Boyle's law.

Examples.

1. Show that if the pressure of the gas at the lower station and the value of g be given, the height at which the density will be halved varies inversely as the density.

2. At what height, in an atmosphere of hydrogen at 0° C., would the density be halved, g being 981?

Ans. $7{\cdot}9954 \times 10^6$.

45. The phenomena of capillarity, soap-bubbles, etc., can be reduced to quantitative expression by assuming a tendency in the surface of every liquid to contract. The following table exhibits the intensity of this contractile force for various liquids at the temperature of 20° C. The contractile force diminishes as the temperature increases.

Superficial tensions at 20° C., *in dynes per linear centim., deduced from Quincke's results.*

	Density.	Tension of Surface separating the Liquid from		
		Air.	Water.	Mercury.
Water,	0·9982	81	0	418
Mercury,	13·5432	540	418	0
Bisulphide of Carbon,	1·2687	32·1	41·75	372·5
Chloroform,	1·4878	30·6	29·5	399
Alcohol,	·7906	25·5	...	399
Olive Oil,	·9136	36·9	20·56	335
Turpentine,	·8867	29·7	11·55	250·5
Petroleum,	·7977	31·7	27·8	284
Hydrochloric Acid,	1·1	70·1	...	377
Solution of Hyposulphite of Soda,	1·1248	77·5	...	442·5

The values here given for water and mercury are only applicable when special precautions are taken to ensure

cleanliness and purity. Without such precautions smaller values will be obtained. (Quincke in *Wied. Ann.*, 1886, page 219.)

The following values are from the observations of A. M. Worthington (*Proc. Roy. Soc.*, June 16, 1881), at temperatures from 15° to 18° C., for surfaces exposed to air :—

Surface Tension.

	In gm. per cm.	In dynes per cm.
Water,	·072 to ·080	70·6 to 78·5
Alcohol,	·02586	25·3
Turpentine,	·02818	27·6
Olive Oil,	·03373	33·1
Chloroform,	·03025	29·6

46. Very elaborate measurements of the thicknesses of soap films have been made by Reinold and Rücker (*Phil. Trans.*, 1881, p. 456; and 1883, p. 651). When so thin as to appear black, the thickness varied from 7·2 to 14·5 millionths of a millimetre, the mean being 11·7. This is $1{\cdot}17 \times 10^{-6}$ centimetre. The following thicknesses were observed for the colours of the successive orders :—

	Thickness. cm.
FIRST ORDER—	
Red,	$2{\cdot}84 \times 10^{-5}$
SECOND ORDER—	
Violet,	3·05 ,,
Blue,	3·53 ,,
Green,	4·09 ,,
Yellow,	4·54 ,,
Orange,	4·91 ,,
Red,	5·22 ,,
THIRD ORDER—	
Purple,	5·59 ,,
Blue,	5·77 ,,
,,	6·03 ,,
Green,	6·56 ,,
Yellow,	$7{\cdot}10 \times 10^{-5}$
Red,	7·65 ,,
Bluish Red,	8·15 ,,
FOURTH ORDER—	
Green,	8·41 ,,
,,	8·93 ,,
Yellow-Green,	9·64 ,,
Red,	10·52 ,,
FIFTH ORDER—	
Green,	$1{\cdot}119 \times 10^{-4}$
,,	1·188 ,,
Red,	1·260 ,,
,,	1·335 ,,

	Thickness. cm.
SIXTH ORDER—	
Green, ...	$1{\cdot}410 \times 10^{-4}$
,, ...	1·479 ,,
Red, ...	1·548 ,,
,, ...	1·627 ,,
SEVENTH ORDER—	
Green, ...	1·705 ,,
Green, ...	$1{\cdot}787 \times 10^{-4}$
Red, ...	1·869 ,,
,, ...	1·936 ,,
EIGHTH ORDER—	
Green, ...	2·004 ,,
Red, ...	2·115 ,,

46A. Depression of the barometrical column due to capillarity, according to Pouillet:—

Internal Diameter of tube.	Depression.	Internal Diameter of tube.	Depression.	Internal Diameter of tube.	Depression.
mm.	mm.	mm.	mm.	mm.	mm.
2	4·579	8·5	·604	15	·127
2·5	3·595	9	·534	15·5	·112
3	2·902	9·5	·473	16	·099
3·5	2·415	10	·419	16·5	·087
4	2·053	10·5	·372	17	·077
4·5	1·752	11	·330	17·5	·068
5	1·507	11·5	·293	18	·060
5·5	1·306	12	·260	18·5	·053
6	1·136	12·5	·230	19	·047
6·5	·995	13	·204	19·5	·041
7	·877	13·5	·181	20	·036
7·5	·775	14	·161	20·5	·032
8	·684	14·5	·143	21	·028

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CHAPTER V.

STRESS, STRAIN, AND RESILIENCE.

47. In the nomenclature introduced by Rankine, and adopted by Thomson and Tait, any change in the shape or size of a body is called a *strain*, and an action of force tending to produce a strain is called a *stress*. We shall always suppose strains to be small; that is, we shall suppose the ratio of the initial to the final length of every line in the strained body to be nearly a ratio of equality.

48. A strain changes every small spherical portion of the body into an ellipsoid; and the strain is said to be homogeneous when equal spherical portions in all parts of the body are changed into equal ellipsoids with their corresponding axes equal and parallel. When the strain consists in change of volume, unaccompanied by change of shape, the ellipsoids are spheres.

When strain is not homogeneous, but varies continuously from point to point, the *strain at any point* is defined by attending to the change which takes place in a very small sphere or cube having the point at its centre, so small that the strain throughout it may be regarded as homogeneous. In what follows we shall suppose strain to be homogeneous, unless the contrary is expressed.

49. The *axes of a strain* are the three directions in the body, at right angles to each other, which coincide with the directions of the axes of the ellipsoids. Lines drawn in the body in these three directions will remain at right angles to each other when the body is restored to its unstrained condition.

A cube with its edges parallel to the axes will be altered by the strain into a *rectangular* parallelepiped. Any other cube will be changed into an oblique parallelepiped.

When the axes have the same directions in space after as before the strain, the strain is said to be *unaccompanied by rotation*. When such parallelism does not exist, the strain is accompanied by rotation, namely, by the rotation which is necessary for bringing the axes from their initial to their final position.

The numbers which specify a strain are mere ratios, and are therefore independent of units.

50. When a body is under the action of forces which strain it, or tend to strain it; if we consider any plane section of the body, the portions of the body which it separates are pushing each other, pulling each other, or exerting some kind of force upon each other, across the section, and the mutual forces so exerted are equal and opposite. The specification of a stress must include a specification of these forces for all sections, and a body is said to be homogeneously stressed when these forces are the same in direction and intensity for all parallel sections. We shall suppose stress to be homogeneous, in what follows, unless the contrary is expressed.

51. When the force-action across a section consists of

a simple pull or push normal to the section, the direction of this simple pull or push (in other words, the normal to the section) is called an *axis of the stress*. A stress (like a strain) has always three axes, which are at right angles to one another. The mutual forces across a section not perpendicular to one of the three axes are in general partly normal and partly tangential—one side of the section is tending to slide past the other.

The force per unit area which acts across any section is called the intensity of the stress on this section, or simply the *stress on this section*. The dimensions of "force per unit area," or $\frac{\text{force}}{\text{area}}$ are $\frac{M}{LT^2}$, which we shall therefore call the *dimensions of stress*.

52. The relation between the stress acting upon a body and the strain produced depends upon the *resilience* of the body, which requires in general 21 numbers for its complete specification. When the body has exactly the same properties in all directions, 2 numbers are sufficient. These specifying numbers are usually called *coefficients of elasticity*; but the word elasticity is used in so many senses that we prefer to call them *coefficients of resilience*. A coefficient of resilience expresses the quotient of a stress (of a given kind) by the strain (of a given kind) which it produces. A highly resilient body is a body which has large coefficients of resilience. Steel is an example of a body with large, and cork of a body with small, coefficients of resilience.

In all cases (for solid bodies) equal and opposite strains (supposed small) require for their production equal and opposite stresses.

53. The coefficients of resilience most frequently referred to are the three following:—

(1) Resilience of volume, or resistence to hydrostatic compression. If V be the original and $V - v$ the strained volume, $\frac{v}{V}$ is called the compression, and when the body is subjected to uniform normal pressure P per unit area over its whole surface, the quotient of P by the compression is the resilience of volume. This is the only kind of resilience possessed by liquids and gases.

(2) Young's modulus, or the longitudinal resilience of a body which is perfectly free to expand or contract laterally. In general, longitudinal extension produces lateral contraction, and longitudinal compression produces lateral extension. Let the unstrained length be L and the strained length $L \pm l$, then $\frac{l}{L}$ is taken as the measure of the longitudinal extension or compression. The stress on a cross section (that is, on a section to which the stress is normal) is called the longitudinal stress, and Young's modulus is the quotient of the longitudinal stress by the longitudinal extension or compression. If a wire of cross section A sq. cm. is stretched with a force of F dynes, and its length is thus altered from L to $L + l$, the value of Young's modulus for the wire is $\frac{F}{A} \cdot \frac{L}{l}$.

(3) "Simple rigidity" or resistance to shearing. This requires a more detailed explanation.

54. A shear may be defined as a strain by which a sphere of radius unity is converted into an ellipsoid of semiaxes 1, $1 + e$, $1 - e$; in other words, it consists of an

extension in one direction combined with an equal compression in a perpendicular direction.

55. A unit square (Fig. 1) whose diagonals coincide with these directions is altered by the strain into a rhombus whose diagonals are $(1+e)\sqrt{2}$ and $(1-e)\sqrt{2}$, and whose area, being half the product of the diagonals, is $1-e^2$, or, to the first order of small quantities, is 1, the same as the area of the original square. The length of a side of the rhombus, being the square root of the

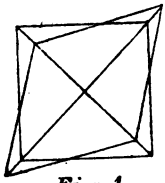

Fig. 1.

Fig. 2

sum of the squares of the semi-diagonals, is found to be $\sqrt{1+e^2}$ or $1+\frac{1}{2}e^2$, and is therefore, to the first order of small quantities, equal to a side of the original square.

56. To find the magnitude of the small angle which a side of the rhombus makes with the corresponding side of the square, we may proceed as follows:—Let *acb* (Fig. 2) be an enlarged representation of one of the small triangles in Fig. 1. Then we have $ab=\frac{1}{2}$, $cb=\frac{1}{2}e\sqrt{2}=\frac{e}{\sqrt{2}}$, angle $cba=\frac{\pi}{4}$. Hence the length of the perpendicular *cd* is $cb\sin\frac{\pi}{4}=\frac{e}{\sqrt{2}}\frac{1}{\sqrt{2}}=\frac{e}{2}$; and since *ad* is ultimately equal to *ab*, we have, to the first order of small quantities,

$$\text{angle } cab = \frac{cd}{ad} = \frac{\frac{1}{2}e}{\frac{1}{2}} = e.$$

The semi-angles of the rhombus are therefore $\frac{\pi}{4} \pm e$, and the angles of the rhombus are $\frac{\pi}{2} \pm 2e$; in other words, each angle of the square has been altered by the amount $2e$. *This quantity $2e$ is adopted as the measure of the shear.*

57. To find the perpendicular distance between opposite sides of the rhombus, we have to multiply a side by the cosine of $2e$, which, to the first order of small quantities, is 1. Hence the perpendicular distance between opposite sides of the square is not altered by the shear, and the relative movement of these sides is represented

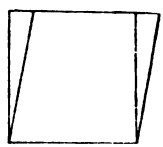

Fig 3.

Fig 4

by supposing one of them to remain fixed, while the other slides in the direction of its own length through a distance of $2e$, as shown in Fig. 3 or Fig. 4. Fig. 3, in fact, represents a shear combined with right-handed rotation, and Fig. 4 a shear combined with left-handed rotation, as appears by comparing these figures with Fig. 1, which represents shear without rotation.

58. The square and rhombus in these three figures may be regarded as sections of a prism whose edges are perpendicular to the plane of the paper, and figures 3 and 4

show that (neglecting rotation) a shear consists in the relative sliding of parallel planes without change of distance, the amount of this sliding being proportional to the distance, and being in fact equal to the product of the distance by the numerical measure of the shear. A good illustration of a shear is obtained by taking a book, and making its leaves slide one upon another.

It may be well to remark, by way of caution, that the selection of the planes is not arbitrary as far as direction is concerned. The only planes which are affected in the manner here described are the two sets of planes which make angles of 45° with the axes of the shear (these axes being identical with the diagonals in Fig. 1).

59. Having thus defined and explained the term "shear," which it will be observed denotes a particular species of *strain*, we now proceed to define a shearing stress.

A *shearing stress* may be defined as the combination of two longitudinal stresses at right angles to each other, these stresses being opposite in sign and equal in magnitude; in other words, it consists of a pull in one direction combined with an equal thrust in a perpendicular direction.

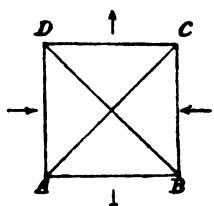

Fig 5.

60. Let P denote the intensity of each of these longitudinal stresses; we shall proceed to calculate the stress upon a plane inclined at 45° to the planes of these stresses. Consider a unit cube so taken that the pull is perpendicular to two of its faces, AB and DC (Fig. 5), and the thrust

is perpendicular to two other faces, AD, BC. The forces which hold the half-cube ABC in equilibrium are—

(1) An outward force P, uniformly distributed over the face AB, and having for its resultant a single force P acting outward applied at the middle point of AB.

(2) An inward force P, having for its resultant a single force P acting inwards at the middle point of BC.

(3) A force applied to the face AC.

To determine this third force, observe that the other two forces meet in a point, namely, the middle point of AC, that their components perpendicular to AC destroy one another, and that their components along AC, or rather along CA, have each the magnitude $\frac{P}{\sqrt{2}}$; hence their resultant is a force $P\sqrt{2}$, tending from C towards A. The force (3) must be equal and opposite to this. Hence each of the two half-cubes ABC, ADC exerts upon the other a force $P\sqrt{2}$, which is tangential to their plane of separation. The stress upon the diagonal plane AC is therefore a purely tangential stress. To compute its intensity we must divide its amount $P\sqrt{2}$ by the area of the plane, which is $\sqrt{2}$, and we obtain the quotient P. Similar reasoning applies to the other diagonal plane BD. P is taken as the *measure of the shearing stress.* The above discussion shows that it may be defined as the *intensity of the stress* either *on the planes of purely normal stress*, or *on the planes of purely tangential stress.*

61. A shearing stress, if applied to a body which has the same properties in all directions (an *isotropic* body), produces a simple shear with the same axes as the stress; for the extension in the direction of the pull will be equal to the compression in the direction of the thrust; and in

the third direction, which is perpendicular to both of these, there is neither extension nor contraction, since the transverse contraction due to the pull is equal to the transverse extension due to the thrust.

A shearing stress applied to a body which has not the same properties in all directions produces in general a shear with the same axes as the stress, combined with some other distortion.

In both cases, the quotient of the shearing stress by the shear produced is called the *resistance to shearing*. In the case of an isotropic body, it is also called the *simple rigidity*.

62. The following values of the resilience of liquids under compression are reduced from those given in Jamin, "Cours de Physique," 2nd edition, tom. i. pp. 168 and 169 :—

	Temp Cent.	Coefficient of Resilience.	Compression for one Atmosphere (megadyne per square centim.)
Mercury, - -	0°·0	$3{\cdot}436 \times 10^{11}$	$2{\cdot}91 \times 10^{-6}$
Water, - -	0·0	$2{\cdot}02 \times 10^{10}$	$4{\cdot}96 \times 10^{-5}$
,, - -	1·5	1·97 ,,	5·08 ,,
,, - -	4·1	2·03 ,,	4·92 ,,
,, - -	10·8	2·11 ,,	4·73 ,,
,, - -	13·4	2·13 ,,	4·70 ,,
,, - -	18·0	2·20 ,,	4·55 ,,
,, - -	25·0	2·22 ,,	4·50 ,,
,, -	34·5	2·24 ,,	4·47 ,,
,, - -	43·0	2·29 ,,	4·36 ,,
,, - -	53·0	2·30 ,,	4·35 ,,
Ether,- - -	0·0	$9{\cdot}2 \times 10^{9}$	$1{\cdot}09 \times 10^{-4}$
	0·0	7·8 ,,	1·29 ,,
	14·0	7·2 ,,	1·38 ,,
Alcohol, - -	7·3	$1{\cdot}22 \times 10^{10}$	8·17 ,,
	13·1	1·12 ,,	$8{\cdot}91 \times 10^{-5}$
Sea Water, - -	17·5	2·33 ,,	4·30 ,,

63. The following are reduced from the results obtained by Amaury and Descamps, "Comptes Rendus," tom. lxviii. p. 1564 (1869), and are probably more accurate than the foregoing, especially in the case of mercury :—

		Coefficient of Resilience.	Compression for one megadyne per square centim.
Distilled Water, -	15°	$2{\cdot}22 \times 10^{10}$	$4{\cdot}51 \times 10^{-5}$
Alcohol, - - -	0	1·21 ,,	8·24 ,,
,, - - -	15	1·11 ,,	8·99 ,,
Ether, - - -	0	$9{\cdot}30 \times 10^{9}$	$1{\cdot}08 \times 10^{-4}$
,, - - -	14	7·92 ,,	1·26 ,,
Bisulphide of Carbon,	14	$1{\cdot}60 \times 10^{10}$	$6{\cdot}26 \times 10^{-5}$
Mercury, - - -	15	$5{\cdot}42 \times 10^{11}$	$1{\cdot}84 \times 10^{-6}$

64. The following values of the coefficients of resilience for solids are reduced from those given in my own papers to the Royal Society (see "Phil. Trans.," Dec. 5th, 1867, p. 369), by employing the value of g at the place of observation, namely, 981·4.

	Young's Modulus.	Simple Rigidity.	Resilience of Volume.	Density
Glass, flint, -	$6{\cdot}03 \times 10^{11}$	$2{\cdot}40 \times 10^{11}$	$4{\cdot}15 \times 10^{11}$	2·942
Another specimen	5·74 ,,	2·35 ,,	3·47 ,,	2·935
Brass, drawn, -	$1{\cdot}075 \times 10^{12}$	3·66 ,,	.....	8·471
Steel, - - -	2·139 ,,	8·19 ,,	$1{\cdot}841 \times 10^{12}$	7·849
Iron, wrought, -	1·963 ,,	7·69 ,,	1·456 ,,	7·677
,, cast, - -	1·349 ,,	5·32 ,,	$9{\cdot}64 \times 10^{11}$	7·235
Copper, - -	1·234 ,,	4·47 ,,	$1{\cdot}684 \times 10^{12}$	8·843

65. The resilience of volume was not directly observed, but was calculated from the values of "Young's modulus" and "simple rigidity," by a formula which is strictly true

for bodies which have the same properties in all directions. The contraction of diameter in lateral directions for a body which is stretched by purely longitudinal stress was also calculated by a formula to which the same remark applies. The ratio of this lateral contraction to the longitudinal extension is called "Poisson's ratio," and the following were its values as thus calculated for the six bodies experimented on :—

Glass, flint,.....	·258	Iron, wrought,.....	·275
Another specimen,	·229	,, cast,.....	·267
Brass, drawn,.....	·469 (?)	Copper, ...	·378
Steel,.....	·310		

Kirchhoff has found for steel the value ·294, and Clerk Maxwell has found for iron ·267. Cornu ("Comptes Rendus," August 2, 1869) has found for different specimens of glass the values ·225, ·226, ·224, ·257, ·236, ·243, ·250, giving a mean of ·237, and maintains (with many other continental savants) that for all isotropic solids (that is, solids having the same properties in all directions) the true value is $\frac{1}{4}$.

66. The following values of Poisson's ratio have been found by Mr. A. Mallock ("Proc. Roy. Soc.," June 19, 1879) :—

Steel, ...	·253	Ivory, ...	·50
Brass,.....	·325	India Rubber,.....	·50
Copper, ...	·348	Paraffin, ...	·50
Lead, ...	·375	Plaster of Paris,....	·181
Zinc (rolled),.....	·180	Cardboard,.....	·2
,, (cast),.....	·230	Cork, ...	·00
Ebonite,.....	·389		

	Radial due to Longitudinal.	Longitudinal due to Radial.	In Cross Section.
Boxwood,.....	·42	·406	
Beechwood, ...	·53	·408	
White Pine,.....	·486	·372	·227

The heading "Radial due to Longitudinal" means that the applied force is longitudinal (that is, parallel to the length of the tree) and that the contraction along a radius of the tree is compared with the longitudinal extension.

67. The following are reduced from Sir W. Thomson's results ("Proc. Roy. Soc.," May, 1865), the value of g being 981·4 :—

	Simple Rigidity.			
Brass, three specimens,.....	4·03	3·48	3·44	$\times 10^{11}$
Copper, two specimens, ...	4·40	4·40		

Other specimens of copper in abnormal states gave results ranging from $3{\cdot}86 \times 10^{11}$ to $4{\cdot}64 \times 10^{11}$.

The following are reduced from Wertheim's results ("Ann. de Chim.," ser. 3, tom. xxiii.), g being taken as 981 :—

Different Specimens of Glass (Crystal).

			$\times 10^{11}$
Young's Modulus,.....	3·41 to 4·34,	mean 3·96	
Simple Rigidity,.....	1·26 to 1·66	,, 1·48	
Volume Resilience,....	3·50 to 4·39	,, 3·89	

Different Specimens of Brass.

			$\times 10^{11}$
Young's Modulus,....	9·48 to 10·44,	mean 9·86	
Simple Rigidity,.....	3·53 to 3·90	,, 3·67	
Volume Resilience,..	10·02 to 10·85	,, 10·43	

68. Savart's experiments on the torsion of brass wire ("Ann. de Chim.," 1829) lead to the value $3{\cdot}61 \times 10^{11}$ for simple rigidity.

Kupffer's values of Young's modulus for nine different specimens of brass range from $7{\cdot}96 \times 10^{11}$ to $11{\cdot}4 \times 10^{11}$, the value generally increasing with the density.

For a specimen, of density 8·4465, the value was $10{\cdot}58 \times 10^{11}$.

For a specimen, of density 8·4930, the value was $11·2 \times 10^{11}$.

The values of Young's modulus found by the same experimenter for steel, range from $20·2 \times 10^{11}$ to $21·4 \times 10^{11}$.

69. The following are reduced from Rankine's "Rules and Tables," pp. 195 and 196, the mean value being adopted where different values are given :—

	Tenacity.		Young's Modulus.
Steel Bars,	$7·93 \times 10^9$		$2·45 \times 10^{12}$
Iron Wire,	5·86 ,,		1·745 ,,
Copper Wire,	4·14 ,,		1·172 ,,
Brass Wire,	3·38 ,,		$9·81 \times 10^{11}$
Lead, Sheet,	$2·28 \times 10^8$		$5·0 \times 10^{10}$
Tin, Cast,	3·17 ,,		.....
Zinc,	5·17 ,,		.....
Ash,	$1·172 \times 10^9$		$1·10 \times 10^{11}$
Spruce,	$8·55 \times 10^8$		1·10 ,,
Oak,	$1·026 \times 10^9$		1·02 ,,
Glass,	$6·48 \times 10^8$		$5·52 \times 10^{11}$
Brick and Cement,	$2·0 \times 10^7$		.....

The tenacity of a substance may be defined as the greatest longitudinal stress that it can bear without tearing asunder. The quotient of the tenacity by Young's modulus will therefore be the greatest longitudinal extension that the substance can bear.

CHAPTER VI.

ASTRONOMY.

Size and Figure of the Earth.

70. ACCORDING to the latest determination, as published by Capt. Clarke in the "Philosophical Magazine" for August, 1878, the semiaxes of the ellipsoid which most nearly agrees with the actual earth are, in feet,

$$a = 20926629,\ b = 20925105,\ c = 20854477,$$

which, reduced to centimetres, are

$$a = 6{\cdot}37839 \times 10^8,\ b = 6{\cdot}37792 \times 10^8,\ c = 6{\cdot}35639 \times 10^8,$$

giving a mean radius of $6{\cdot}3709 \times 10^8$, and a volume of $1{\cdot}0832 \times 10^{27}$ cubic centims.

The ellipticities of the two principal meridians are

$$\frac{1}{289{\cdot}5} \text{ and } \frac{1}{295{\cdot}8}.$$

The longitude of the greatest axis is 8° 15′ W The mean length of a quadrant of the meridian is $1{\cdot}00074 \times 10^9$.

The length of a minute of latitude is approximately 185200 – 940 cos. 2 lat. of middle of arc.

The mass of the earth, assuming Baily's value 5·67 for the mean density, is $6{\cdot}14 \times 10^{27}$ grammes.

Day and Year.

Sidereal day,	86164 mean solar seconds.
Sidereal year,	31,558,150 ,,
Tropical year,	31,556,929 ,,
Angular velocity of earth's rotation,	$\frac{2\pi}{86164} = \frac{1}{13713}$.
Velocity of points on the equator due to earth's rotation,	46510 centims. per second.
Velocity of earth in orbit, about	2960600 ,,
Centrifugal force at equator due to earth's rotation,	3·3908 dynes per gramme.

Attraction in Astronomy.

71. The mass of the moon is the product of the earth's mass by ·011364, and is therefore to be taken as $6{\cdot}98 \times 10^{25}$ grammes, the doubtful element being the earth's mean density, which we take as 5·67.

The mean distance of the centres of gravity of the earth and moon is 60·2734 equatorial radii of the earth —that is, $3{\cdot}8439 \times 10^{10}$ centims.

The mean distance of the sun from the earth is about $1{\cdot}487 \times 10^{13}$ centims., or 92·39 million miles, corresponding to a parallax of 8″·848.*

The intensity of centrifugal force due to the earth's motion in its orbit (regarded as circular) is $\left(\frac{2\pi}{T}\right)^2 r$, r denoting the mean distance, and T the length of the sidereal year, expressed in seconds. This is equal to the acceleration due to the sun's attraction at this distance. Putting for r and T their values, $1{\cdot}487 \times 10^{13}$ and $3{\cdot}1558 \times 10^{7}$, we have

$$\left(\frac{2\pi}{T}\right)^2 r = {\cdot}5894.$$

* This value of the mean solar parallax was determined by Professor Newcomb, and was adopted in the "Nautical Almanac" for 1882. (See Art. 86 for a later determination.)

This is about $\frac{1}{1660}$ of the value of g at the earth's surface.

The intensity of the earth's attraction at the mean distance of the moon is about

$$\frac{981}{(60{\cdot}27)^2} \text{ or } {\cdot}2701.$$

This is less than the intensity of the sun's attraction upon the earth and moon, which is ·5894 as just found. Hence the moon's path is always concave towards the sun.

72. The mutual attractive force F between two masses m and m', at distance l, is

$$F = C\frac{mm'}{l^2}$$

where C is a constant. To determine its value, consider the case of a gramme at the earth's surface, attracted by the earth. Then we have

$$F = 981,\ m = 1,\ m' = 6{\cdot}14 \times 10^{27},\ l = 6{\cdot}37 \times 10^8;$$

whence we find

$$C = \frac{6{\cdot}48}{10^8} = \frac{1}{1{\cdot}543 \times 10^7}.$$

To find the mass m which, at the distance of 1 centim. from an equal mass, would attract it with a force of 1 dyne, we have $1 = Cm^2$;

whence $$m = \sqrt{\frac{1}{C}} = 3928 \text{ grammes.}$$

73. To find the acceleration a produced at the distance of l centims. by the attraction of a mass of m grammes, we have $$a = \frac{F}{m'} = C\frac{m}{l^2},$$

where C has the value $6{\cdot}48 \times 10^{-8}$ as above.

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To find the dimensions of C we have $C = \frac{l^2 a}{m}$, where the dimensions of a are LT^{-2}.

The dimensions of C are therefore

$$L^2 M^{-1} LT^{-2}; \text{ that is, } L^3 M^{-1} T^{-2}.$$

74. The equation $a = C\frac{m}{l^2}$ shows that when $a = 1$ and $l = 1$, m must equal $\frac{1}{C}$; that is to say, the mass which produces unit acceleration at the distance of 1 centimetre is $1{\cdot}543 \times 10^7$ grammes. If this were taken as the unit of mass, the centimetre and second being retained as the units of length and time, the acceleration produced by the attraction of any mass at any distance would be simply the quotient of the mass by the square of the distance.

It is thus theoretically possible to base a general system of units upon two fundamental units alone; one of the three fundamental units which we have hitherto employed being eliminated by means of the equation

$$\text{mass} = \text{acceleration} \times (\text{distance})^2,$$

which gives for the dimensions of M the expression $L^3 T^{-2}$.

Such a system would be eminently convenient in astronomy, but could not be applied with accuracy to ordinary terrestrial purposes, because we can only roughly compare the earth's mass with the masses which we weigh in our balances.

75. The mass of the earth on this system is the product of the acceleration due to gravity at the earth's surface, and the square of the earth's radius. This product is

$$981 \times (6{\cdot}37 \times 10^8)^2 = 3{\cdot}98 \times 10^{20},$$

and is independent of determinations of the earth's density.

The new unit of force will be the force which, acting upon the new unit of mass, produces unit acceleration. It will therefore be equal to $1{\cdot}543 \times 10^7$ dynes; and its dimensions will be

$$\text{mass} \times \text{acceleration} = (\text{acceleration})^2 \times (\text{distance})^2$$
$$= L^4 T^{-4}.$$

76. If we adopt a new unit of length equal to l centims., and a new unit of time equal to t seconds, while we define the unit mass as that which produces unit acceleration at unit distance, the unit mass will be

$$l^3 t^{-2} \times 1{\cdot}543 \times 10^7 \text{ grammes.}$$

If we make l the wave-length of the line F in vacuo, say,

$$4{\cdot}86 \times 10^{-5},$$

and t the period of vibration of the same ray, so that $\frac{l}{t}$ is the velocity of light in vacuo, say,

$$3 \times 10^{10},$$

the value of $l^3 t^{-2}$ or $l\left(\frac{l}{t}\right)^2$ is

$$4{\cdot}374 \times 10^{16},$$

and the unit mass will be the product of this quantity into $1{\cdot}543 \times 10^7$ grammes. This product is $6{\cdot}75 \times 10^{23}$ grammes.

The mass of the earth in terms of this unit is

$$3{\cdot}98 \times 10^{20} \div (4{\cdot}374 \times 10^{16}) = 9100,$$

and is independent of determinations of the earth's density.

CHAPTER VII.

VELOCITY OF SOUND.

77. THE propagation of sound through any medium is due to the elasticity or resilience of the medium; and the general formula for the velocity of propagation s is

$$s = \sqrt{\frac{E}{D}},$$

where D denotes the density of the medium, and E the coefficient of resilience.

78. For air, or any gas, we are to understand by E the quotient

$$\frac{\text{increment of pressure}}{\text{corresponding compression}};$$

that is to say, if P, $P + p$ be the initial and final pressures, and V, $V - v$ the initial and final volumes, p and v being small in comparison with P and V, we have

$$E = \frac{p}{\frac{v}{V}} = p\frac{V}{v}.$$

If the compression took place at constant temperature, we should have

$$\frac{p}{P} = \frac{v}{V} \text{ and } E = P.$$

But in the propagation of sound, the compression is effected so rapidly that there is not time for any sensible part of the heat of compression to escape, and we have

$$\frac{p}{P}=\gamma\frac{v}{V},\quad E=\gamma P,\quad s=\sqrt{\gamma\frac{P}{D}},$$

where $\gamma=1{\cdot}41$ for dry air, oxygen, nitrogen, or hydrogen.

The value of $\frac{P}{D}$ for dry air at $t°$ Cent. (see p. 46) is

$$(1+{\cdot}00366t)\times 7{\cdot}838+10^8.$$

Hence the velocity of sound through dry air is

$$s=10^4\sqrt{1{\cdot}41\times(1+{\cdot}00366t)\times 7{\cdot}838}$$
$$=33240\sqrt{1+{\cdot}00366t};$$

or approximately, for atmospheric temperatures,

$$s=33240+60t.$$

79. In the case of any liquid, E denotes the resilience of volume.*

For water at 8°·1 C. (the temperature of the Lake of Geneva in Colladon's experiment) we have

$$E=2{\cdot}08\times10^{10},\ D=1 \text{ sensibly};$$
$$\therefore\quad \sqrt{\frac{E}{D}}=\sqrt{E}=144000,$$

the velocity as determined by Colladon was 143500.

80. For the propagation of sound along a solid, in the form of a thin rod, wire, or pipe, which is free to expand or contract laterally, E must be taken as denoting Young's modulus of elasticity.* The values of E and D will be different for different specimens of the same material. Employing the values given in the Table (§ 64), we have

* Strictly speaking, E should be taken as denoting the resilience for sudden applications of stress—so sudden that there is not time for changes of temperature produced by the stress to be sensibly diminished by conduction. This remark applies to both §§ 79 and 80. For the amount of these changes of temperature, see a later section under Heat.

	Values of E.	Values of D.	Values of $\sqrt{\frac{E}{D}}$, or velocity.
Glass, first specimen,	$6{\cdot}03 \times 10^{11}$	2·942	$4{\cdot}53 \times 10^5$
,, second specimen,	5·74	2·935	4·42 ,,
Brass, - - -	$1{\cdot}075 \times 11^{12}$	8·471	3·56 ,,
Steel, - - -	2·139 ,,	7·849	5·22 ,,
Iron, wrought, - -	1·963 ,,	7·677	5·06 ,,
,, cast, - - -	1·349 ,,	7·235	4·32 ,,
Copper, - - -	1·234 ,,	8·843	3·74 ,,

81. If the density of a specimen of red pine be ·5, and its modulus of longitudinal elasticity be $1{\cdot}6 \times 10^6$ pounds per square inch at a place where g is 981, compute the velocity of sound in the longitudinal direction.

By the table of stress, page 4, a pound per square inch (g being 981) is $6{\cdot}9 \times 10^4$ dynes per square centim. Hence we have for the required velocity

$$\sqrt{\frac{E}{D}} = \sqrt{\frac{1{\cdot}6 \times 10^6 \times 6{\cdot}9 \times 10^4}{{\cdot}5}} = 4{\cdot}7 \times 10^5$$

centims. per second.

82. The following numbers, multiplied by 10^5, are the velocities of sound through the principal metals, as determined by Wertheim:—

	At 20° C.	At 100° C.	At 200° C.
Lead, - - -	1·23	1·20	...
Gold, - - -	1·74	1·72	1·73
Silver, - - -	2·61	2·64	2·48
Copper, - - -	3·56	3·29	2·95
Platinum, - -	2·69	2·57	2·46
Iron, - - -	5·13	5·30	4·72
Iron Wire (ordinary),	4·92	5·10	...
Cast Steel, - -	4·99	4·92	4·79
Steel Wire (English),	4·71	5·24	5·00
,, - -	4·88	5·01	...

The following velocities in wood are from the observations of Wertheim and Chevandier, "Comptes Rendus," 1846, pp. 667 and 668:—

	Along Fibres.	Radial Direction.	Tangential Direction.
Pine, - - -	$3{\cdot}32 \times 0^5$	$2{\cdot}83 \times 10^5$	$1{\cdot}59 \times 10^5$
Beech, - - -	3·34 ,,	3·67 ,,	2·83 ,,
Witch-Elm, - -	3·92 ,,	3·41 ,,	2·39 ,,
Birch, - - -	4·42 ,,	2·14 ,,	3·03 ,,
Fir, - - -	4·64 ,,	2·67 ,,	1·57 ,,
Acacia, - - -	4·71 ,,		
Aspen, - - -	5·08 ,,		

Musical Strings.

83. Let M denote the mass of a string per unit length,

F ,, stretching force,

L ,, length of the vibrating portion;

then the velocity with which pulses travel along the string is

$$v = \sqrt{\frac{F}{M}},$$

and the number of vibrations made per second is

$$n = \frac{v}{2L}.$$

Example.

For the four strings of a violin the values of M in grammes per centimetre of length are

·00416, ·00669, ·0106, ·0266.

The values of n are

660, 440, $293\frac{1}{3}$, $195\frac{5}{9}$;

and the common value of L is 33 centims. Hence the values of v or $2Ln$ are

43560, 29040, 19360, 12910

centims. per second; and the values of F or Mv^2, in dynes, are

$7{\cdot}89 \times 10^6$, $5{\cdot}64 \times 10^6$, $3{\cdot}97 \times 10^6$, $4{\cdot}43 \times 10^6$.

Faintest Audible Sound.

84. Lord Rayleigh ("Proc. R. S.," 1877, vol. xxvi. p. 248), from observing the greatest distance at which a whistle giving about 2730 vibrations per second, and blown by water-power, was audible without effort in the middle of a fine still winter's day, calculates that the maximum velocity of the vibrating particles of air at this distance from the source was ·0014 centims. per second, and that the amplitude was $8{\cdot}1 \times 10^{-8}$ centims., the calculation being made on the supposition that the sound spreads uniformly in hemispherical waves, and no deduction being made for dissipation, nor for waste energy in blowing.

CHAPTER VIII.

LIGHT.

85. ALL kinds of light are believed to have the same velocity in vacuo. The velocity of light of given refrangibility in any medium is $\frac{1}{\mu}$ of its velocity in vacuo, μ denoting the absolute index of refraction of that medium for light of the given refrangibility.

Light of given refrangibility is light of given wave-frequency. Its wave-length in any medium is the quotient of its velocity in that medium by its wave-frequency. If n denote the wave-frequency (that is to say, the number of waves which traverse a given point in a second), the wave-length in any medium will be $\frac{1}{n\mu}$ of the velocity in vacuo.

The absolute index of refraction for ordinary air is about 1·00029. More accurate statements of its value will be found in Arts. 94–96.

86. The best determination of the velocity of light is that made by Professor Newcomb at Washington in 1882 ("Astron. Papers of Amer. Ephem.," vol. ii. parts iii. and iv. 1885). The method employed was that of the revolving mirror, the distance between the revolving and

the fixed mirror being in one portion of the observations 2550 metres, and in the remaining portion 3720 metres. The resulting velocity *in vacuo* is

$$2{\cdot}99860 \times 10^{10} \text{ centims. per sec.}$$

The following summary of results is from Professor Newcomb's paper, page 202 :—

	km. per. sec.
Michelson, at Naval Academy, in 1879, ...	299910
Michelson, at Cleveland, 1882, ...	299853
Newcomb, at Washington, 1882, using only results supposed to be nearly free from constant errors, ...	299860
Newcomb, including all determinations, ...	299810
Foucault, at Paris, in 1862, ...	298000
Cornu, at Paris, in 1874, ...	298500
Cornu, at Paris, in 1878, ...	300400
This last result as discussed by Listing, ...	299990
Young and Forbes, 1880-81, ...	301382

Professor Newcomb remarks (page 203) that the value 299860 km. per sec. for the velocity of light, combined with Clark's value 6378·2 km. for the earth's equatorial radius, and Nyren's value 20″·492 for the constant of aberration, gives for the solar parallax the value 8″·794.

87. The following are the wave-lengths adopted by Ångström for the principal Fraunhofer lines in air at 760 millims. pressure (at Upsal) and 16° C. :—

		Centims.
	A	$7{\cdot}604 \times 10^{-5}$
	B	$6{\cdot}867 \times 10^{-5}$
	C	$6{\cdot}56201 \times 10^{-5}$
Mean of lines	D	$5{\cdot}89212 \times 10^{-5}$
	E	$5{\cdot}26913 \times 10^{-5}$
	F	$4{\cdot}86072 \times 10^{-5}$
	G	$4{\cdot}30725 \times 10^{-5}$
	H_1	$3{\cdot}96801 \times 10^{-5}$
	H_2	$3{\cdot}93300 \times 10^{-5}$

These numbers will be approximately converted into the corresponding wave-lengths in vacuo by multiplying them by 1·00029.

88. Assuming 3×10^{10} to be the velocity of light in air, and neglecting the difference of velocity between the more and less refrangible rays, we obtain the following frequencies by dividing the common velocity by Ångström's values of the wave-lengths :—

	Vibrations per Second.
A	$3{\cdot}945 \times 10^{14}$
B	4·369 ,,
C	4·572 ,,
D	5·092 ,,
E	5·693 ,,
F	6·172 ,,
G	6·965 ,,
H_1	7·560 ,,
H_2	7·628 ,,

According to Langley ("Com. Ren.," Jan., 1886), the solar spectrum extends beyond the red as far as wave-length 27×10^{-5}, and the radiation from terrestrial bodies at temperatures below 100° extends as far as wave-length 150×10^{-5}. The frequencies corresponding to these two wave-lengths are $1{\cdot}1 \times 10^{14}$ and 2×10^{13}.

INDICES OF REFRACTION OF SOLIDS.

89. Dr. Hopkinson ("Proc. R. S.," June 14, 1877) has determined the indices of refraction of the principal varieties of optical glass made by Messrs. Chance, for the fixed lines A, B, C, D, E, *b*, F, (G), G, *h*, H_1. By D is to be understood the more refrangible of the pair of sodium lines; by *b* the most refrangible of the group of magnesium lines; by (G) the hydrogen line near G.

In connection with the results of observation, he employs the empirical formula

$$\mu - 1 = a\{1 + bx(1 + cx)\},$$

where x is a numerical name for the definite ray of which μ is the refractive index. In assigning the value of x, four glasses—hard crown, soft crown, light flint, and dense flint—were selected on account of the good accord of their results; and the mean of their indices for any given ray being denoted by $\bar{\mu}$, the value assigned to x for this ray is $\bar{\mu} - \bar{\mu}_F$ where $\bar{\mu}_F$ denotes the value of $\bar{\mu}$ for the line F.

The value of $\bar{\mu}$ as a function of λ, the wave-length in 10^{-4} centimetres, was found to be approximately

$$\bar{\mu} = 1{\cdot}538414 + 0{\cdot}0067669\frac{1}{\lambda^2} - 0{\cdot}0001734\frac{1}{\lambda^4} + 0{\cdot}000023\frac{1}{\lambda^6}.$$

The following were the results obtained for the different specimens of glass examined:—

Hard Crown, 1st specimen, density 2·48575.

$a = 0{\cdot}523145$, $b = 1{\cdot}3077$, $c = -2{\cdot}33$.

Means of observed values of μ.

A 1·511755;	B 1·513624;	C 1·514571;	D 1·517116;
E 1·520324;	*b* 1·520962;	F 1·523145;	(G) 1·527996;
G 1·528348;	*h* 1·530904;	H_1 1·532789.	

Soft Crown, density 2·55035.

$a = 0{\cdot}5209904$, $b = 1{\cdot}4034$, $c = -1{\cdot}58$.

Means of observed values of μ.

A 1·508956;	B 1·510918;	C 1·511910;	D 1·514580;
E 1·518017;	*b* 1·518678;	F 1·520994;	(G) 1·526208;
G 1·526592;	*h* 1·529360;	H_1 1·531415.	

Extra Light Flint Glass, density 2·86636.

$a = 0{\cdot}549123$, $b = 1{\cdot}7064$, $c = -0{\cdot}198$.

Means of observed values of μ.

A 1·534067;	B 1·536450;	C 1·537682;
D 1·541022;	E 1·545295;	*b* 1·546169;
F 1·549125;	(G) 1·555870;	G 1·556375;
h 1·559992;	H_1 1·562760.	

Light Flint Glass, density 3·20609.

$a = 0{\cdot}583887$, $b = 1{\cdot}9605$, $c = 0{\cdot}53$.

Means of observed values of μ.

B 1·568558;	C 1·570007;	D 1·574013;
E 1·579227;	*b* 1·580273;	F 1·583881;
(G) 1·592184;	G 1·592825;	*h* 1·597332;
H_1 1·600717.		

Dense Flint, density 3·65865.

$a = 0{\cdot}634744$, $b = 2{\cdot}2694$, $c = 1{\cdot}48$.

Means of observed values of μ.

B 1·615704;	C 1·617477;	D 1·622411;
E 1·628882;	*b* 1·630208;	F 1·634748;
(G) 1·645268;	G 1·646071;	*h* 1·651830;
H_1 1·656229.		

Extra Dense Flint, density 3·88947.

$a = 0{\cdot}664226$, $b = 2{\cdot}4446$, $c = 1{\cdot}87$.

Means of observed values of μ.

A 1·639143;	B 1·642894;	C 1·644871;
D 1·650374;	E 1·657631;	*b* 1·659108;
F 1·664246;	(G) 1·676090;	G 1·677020;
h 1·683575;	H_1 1·688590.	

Double Extra Dense Flint, density 4·42162.

$a=0·727237$, $b=2·7690$, $c=2·70$.

Means of observed values of μ.

A 1·696531;	B 1·701080;	C 1·703485;
D 1·710224;	E 1·719081;	*b* 1·720908;
F 1·727257;	(G) 1·742058;	G 1·743210;
h 1·751485.		

90. The following indices of rock salt, sylvin, and alum for the chief Fraunhofer lines are from Stefan's observations:—

	Rock Salt at 17° C.	Sylvin at 20° C.	Alum at 21° C.
A	1·53663	1·48377	1·45057
B	·53918	·48597	·45262
C	·54050	·48713	·45359
D	·54418	·49031	·45601
E	·54901	·49455	·45892
F	·55324	·49830	·46140
G	·56129	·50542	·46563
H	·56823	·51061	·46907

91. Indices of other singly refracting solids—

	Index of Refraction.	Kind of Light.	Observer.
Diamond, ...	2·470	D	Schrauf.
Fluor-spar, ...	1·4339	D	Stefan.
Amber, ...	1·532	D	Kohlrausch.
Rosin, ...	1·545	Red	Jamin.
Copal, ...	1·528	Red	,,
Gum Arabic, ...	1·480	Red	,,
Peru Balsam, ...	1·593	D	Baden Powell.
Canada Balsam, .	1·528	Red	Wollaston.

Effect of Temperature.

According to Stefan, the index of refraction of glass *increases* by about ·000002 for each degree Cent. of

increase of temperature, and the index of rock salt *diminishes* by about ·000 037 for each degree of increase of temperature.

92. Doubly refracting crystals:—

Uniaxal Crystals.

	Ordinary Index.	Extraordinary Index.	Kind of Light.	Temp.	Observer.
Ice, ...	1·3060	1·3073	Red		Reusch.
Iceland-spar,....	1·65844	1·48639	D	24°	v. d. Willigen.
Nitrate of Soda,	1·5854	1·3369	D	23°	F. Kohlrausch.
Quartz, ...	1·54419	1·55329	D	24°	v. d. Willigen.
Tourmaline,....	1·6479	1·6262	Green	22°	Heusser.
Zircon, ...	1·92	1·97	Red		de Senarmont.

Biaxal Crystals.

THREE PRINCIPAL INDICES OF REFRACTION FOR SODIUM LIGHT.

	Least.	Intermediate.	Greatest.	Temp.	Observer.
Arragonite,	1·53013	1·68157	1·68589		Rudberg.
Borax,	1·4463	1·4682	1·4712	23°	Kohlrausch.
Mica,	1·5809	1·5941	1·5997	23°	,,
Nitre,	1·3346	1·5056	1·5064	16°	Schrauf.
Selenite,	1·52082	1·52287	1·53048	17°	v. Lang.
Sulphur (prismatic)	1·9505	2·0383	2·2405	16°	Schrauf.
Topaz,	1·61161	1·61375	1·62109		Rudberg.

INDICES OF REFRACTION FOR LIQUIDS.

93. The following values of indices of refraction for liquids are condensed from Fraunhofer's determinations, as given by Sir John Herschel ("Enc. Met. Art.," *Light*, p. 415):—

Water, density 1·000.

B 1·3309; C 1·3317; D 1·3336;
E 1·3358; F 1·3378; G 1·3413;
H 1·3442.

Oil of Turpentine, density 0·885.

B 1·4705;	C 1·4715;	D 1·4744;	E 1·4784;
F 1·4817;	G 1·4882;	H 1·4939.	

The following determinations of the refractive indices of liquids are from Gladstone and Dale's results, as given in Watt's "Dictionary of Chemistry," iii. pp. 629-631 :—

Sulphide of Carbon, at temperature 11°.

A 1·6142;	B 1·6207;	C 1·6240;	D 1·6333;
E 1·6465;	F 1·6584;	G 1·6836;	H 1·7090.

Benzene, at temperature 10·5°.

A 1·4879;	B 1·4913;	C 1·4931;	D 1·4975;
E 1·5036;	F 1·5089;	G 1·5202;	H 1·5305.

Chloroform, at temperature 10°.

A 1·4438;	B 1·4457;	C 1·4466;	D 1·4490;
E 1·4526;	F 1·4555;	G 1·4614;	H 1·4661.

Alcohol, at temperature 15°.

A 1·3600;	B 1·3612;	C 1·3621;	D 1·3638;
E 1·3661;	F 1·3683;	G 1·3720;	H 1·3751.

Ether, at temperature 15°.

A 1·3529;	B 1·3545;	C 1·3554;	D 1·3566;
E 1·3590;	F 1·3606;	G 1·3646;	H 1·3683.

Water, at temperature 15°.

A 1·3284;	B 1·3300;	C 1·3307;	D 1·3324;
E 1·3347;	F 1·3366;	G 1·3402;	H 1·3431.

INDICES FOR GASES.

94. Indices of refraction of air at 0° C. and 760 mm. for the principal fraunhofer lines.

	According to Kettler.	According to Lorenz.
A	1·00029286	1·00028935
B	29350	28993
C	29383	29024
D	29470	29108
E	29584	29217
F	29685	29312
G	29873	29486
H	30026	29631

95. The formula established by the experiments of Biot and Arago for the index of refraction of air at various pressures and temperatures was

$$\mu - 1 = \frac{\cdot 0002943}{1 + at} \cdot \frac{h}{760},$$

a denoting the coefficient of expansion ·00366, and h the pressure in millims. of mercury at zero. As the pressure of 760 millims. of such mercury at Paris is $1{\cdot}0136 \times 10^6$ dynes per sq. cm., the general formula applicable to all localities alike will be

$$\mu - 1 = \frac{\cdot 0002943}{1 + \cdot 00366t} \cdot \frac{\mathrm{P}}{1{\cdot}0136 \times 10^6},$$

where P denotes the pressure in dynes per sq. cm. This can be reduced to the form

$$\mu - 1 = \frac{\cdot 0002903}{1 + \cdot 00366t} \cdot \frac{\mathrm{P}}{10^6}.$$

96. According to Mascart, $\mu - 1$ for any gas is proportional not to $\frac{h}{1 + at}$ but to

$$\frac{h + \beta h^2}{1 + a't},$$

where β and a' are coefficients which vary from one gas to another. In the following table, the column headed μ_0

contains the indices for 0° and 760 mm. at Paris. The next column contains the value of β multiplied by 10^7 (it being understood that h is expressed in millimetres), and the next column the value of a'. All these data are for the light of a sodium flame :—

	μ_0	$\beta \times 10^7$	a'
Air, ...	1·0002927	7·2	·00382
Nitrogen, ...	2977	8·5	382
Oxygen, ...	2706	11·1	.....
Hydrogen, ...	1387	−8·6	378
Nitrous Oxide, ...	5159	88	388
Nitrous Gas, ...	2975	7	367
Carbonic Oxide, ...	3350	8·9	367
Carbonic Acid, ...	4544	72	406
Sulphurous Acid, ...	7036	25	460
Cyanogen, ...	8216	27·7	.....

More recent, and probably more accurate observations, which will be published in vol. v. of "Travaux et Mémoires du Bureau International des Poids et Mesures," have been conducted by Benoît with Fizeau's dilatometer. They give

1·0002923

as the index of refraction of air for the D line at 0° C. and 760 mm.; and for the temperature coefficient they give

·003667,

which is identical with the coefficient of expansion of air. The larger value, ·00382, obtained by Mascart, is traced to imperfect measurement of temperature.

Coefficient of Dispersive Power.

97. Assuming Cauchy's formula

$$\mu - 1 = a\left(1 + \frac{b}{\lambda^2}\right)$$

(where λ is the wave-length), which is known to be approximately true for air within the limits of the visible spectrum, the constant b may be called the *coefficient of dispersive power*. Employing as the unit of length for λ the 10^{-4} of a centimetre, Mascart ("Ann. de l' École Normale," 1877, p. 62) has obtained the following values for b:—

	Coefficient of Dispersion.
Air,	·0058
Nitrogen,	·0067
Oxygen,	·0064
Hydrogen,	·0043
Carbonic Oxide,	·0075
Carbonic Acid,	·0052
Nitrous Oxide,	·0125
Cyanogen,	·0100

According to Mascart, the ratio of dispersion to deviation for the two lines B and H is ·024 for air, ·032 for the ordinary ray in quartz, ·038 for light crown glass, ·040 for water, and ·046 for the ordinary ray in Iceland-spar.

Rotation of Plane of Polarization.

98. The rotation produced by 1 millim. of thickness of quartz cut perpendicular to the axis has the following values for different portions of the spectrum, according to the observations of Soret and Sarasin ("Com. Ren. 95," p. 635, 1882), the temperature of the quartz being 20° C.:—

	Rotation.		Rotation.
A	12°·668	E	27°·543
B	15°·746	F	32°·773
C	17°·318	G	42°·604
D_2	21°·684	H	51°·193
D_1	21°·727		

According to the same observers, the rotation at $t°$ C. is equal to the rotation at 0° C. multiplied by $1 + \cdot 000179t$.

Units of Illuminating Power.

99. The British "Candle" is a spermaceti candle, $\frac{7}{8}$ inch in diameter (6 to the lb.), burning 120 grains per hour.

The French "Carcel" is a lamp of specified construction, burning 42 grammes of pure Colza oil per hour. One "carcel" is equal to about $9\frac{1}{2}$ "candles."

The unit adopted by the International Congress at Paris, April 1884, is a square centimetre of molten platinum at the temperature of solidification. The surface illuminated by it in photometric tests is to be normally opposite to the surface of the molten platinum. According to the experiments of M. Violle the author of this unit, it is equal to 2·08 carcels. It is therefore about $19\frac{3}{4}$ candles.

CHAPTER IX.

HEAT.

100. THE unit of heat is usually defined as the quantity of heat required to raise, by one degree, the temperature of unit mass of water, initially at a certain standard temperature. The standard temperature usually employed is 0° C.; but this is liable to the objection that ice may be present in water at this temperature. Hence 4° C. has been proposed as the standard temperature; and another proposition is to employ as the unit of heat one hundredth part of the heat required to raise the unit mass of water from 0° to 100° C.

101. According to Regnault ("Mém. Acad. Sciences," xxi. p. 729) the quantity of heat required to raise a given mass of water from 0° to $t°$ C. is proportional to

$$t + \cdot 000\ 02t^2 + \cdot 000\ 000\ 3t^3. \quad . \quad . \quad . \quad (1)$$

The *mean thermal capacity* of a body *between two stated temperatures* is the quantity of heat required to raise it from the lower of these temperatures to the higher, divided by the difference of the temperatures. The mean thermal capacity of a given mass of water between 0° C. and $t°$ is therefore proportional to

$$1 + \cdot 000\ 02t + \cdot 000\ 000\ 3t^2. \quad . \quad . \quad . \quad (2)$$

The *thermal capacity* of a body *at a stated temperature* is the limiting value of the mean thermal capacity as the range is indefinitely diminished. Hence the thermal capacity of a given mass of water at $t°$ is proportional to the differential coefficient of (1), that is to

$$1 + \cdot 000\ 04t + \cdot 000\ 000\ 9t^2. \quad . \quad . \quad . \quad (3)$$

Hence the thermal capacities at 0° and 4° are as 1 to 1·000174 nearly; and the thermal capacity at 0° is to the mean thermal capacity between 0° and 100° as 1 to 1·005.

102. If we agree to adopt the capacity of unit mass of water at a stated temperature as the unit of capacity, the unit of heat must be defined as n times the quantity of heat required to raise unit mass of water from this initial temperature through $\frac{1}{n}$ of a degree when n is indefinitely great.

Supposing the standard temperature and the length of the degree of temperature to be fixed, the units both of heat and of thermal capacity vary directly as the unit of mass.

In what follows, we adopt as the unit of heat (except where the contrary is stated) the heat required to raise a gramme of pure water through 1° C. at a temperature intermediate between 0° and 4°. This specification is sufficiently precise for the statement of any thermal measurements hitherto made.

103. The *thermal capacity of unit mass* of a substance at any temperature is called the *specific heat* of the substance at that temperature

Specific heat is of zero dimensions in length, mass, and time. It is in fact the ratio

$$\frac{\text{increment of heat in the substance}}{\text{increment of heat in water}}$$

for a given increment of temperature, the comparison being between *equal masses* of the substance at the actual temperature and of water at the standard temperature. The numerical value of a given concrete specific heat merely depends upon the standard temperature at which the specific heat of water is called unity.

104. The *thermal capacity of unit volume* of a substance is another important element: we shall denote it by c. Let s denote the specific heat, and d the density of the substance; then c is the thermal capacity of d units of mass, and therefore $c = sd$. The dimensions of c in length, mass, and time are the same as those of d, namely, $\frac{M}{L^3}$. Its numerical value will not be altered by any change in the units of length, mass, and time, which leaves the value of the density of water unchanged.

In the C.G.S. system, since the density of water between 0° and 4° is very approximately unity, the thermal capacity of unit volume of a substance is the value of the ratio

$$\frac{\text{increment of heat in the substance}}{\text{increment of heat in water}}$$

for a given increment of temperature, when the comparison is between *equal volumes.*

105. Mr. Herbert Tomlinson ("Proc. Roy. Soc.," June 19, 1885) has obtained the following determinations of specific heat from observations conducted in a uniform

manner with metallic wires well annealed. The wires were heated sometimes to 60° C. and sometimes to 100° C., and were plunged in water at 20°. The formulæ are for the true specific heat at $t°$ C :—

Aluminium,	$\cdot20700 + \cdot000\,2304t$
Copper,	$\cdot09008 + \cdot000\,0648t$
German Silver,	$\cdot09413 + \cdot000\,0106t$
Iron,	$\cdot10601 + \cdot000\,140t$
Lead,	$\cdot02998 + \cdot000\,031t$
Platinum,	$\cdot03198 + \cdot000\,013t$
Platinum Silver,	$\cdot04726 + \cdot000\,028t$
Silver,	$\cdot05466 + \cdot000\,044t$
Tin,	$\cdot05231 + \cdot000\,072t$
Zinc,	$\cdot09009 + \cdot000\,075t$

The formulæ for the mean specific heat between 0° and $t°$ are obtained from these by leaving the first term unchanged and halving the second term.

Violle has made the following determinations of specific heat at $t°$:—

Platinum,	$\cdot0317 + \cdot000\,012t$
Iridium,	$\cdot0317 + \cdot000\,012t$
Palladium,	$\cdot0582 + \cdot000\,020t$

H. F. Weber has determined the specific heat of diamond to be

$$\cdot0947 + \cdot000\,994t - \cdot000\,000\,36t^2,$$

and consequently the mean specific heat of diamond from 0° to $t°$ to be

$$\cdot0947 + \cdot000\,497t - \cdot000\,000\,12t^2.$$

The mean specific heat of ice according to Regnault is ·504 between − 20° and 0°, and ·474 between − 78° and 0°.

106. The following list of specific heats of elementary substances is condensed from that given in Landolt and Börnstein's tables :—

Substance.	Temperature.	Sp. Heat.	Observer.
Aluminium,	15° to 97°	·2122	Regnault.
Antimony,	13 ,, 106	·0486	Bède.
Arsenic (crystalline),	21 ,, 68	·0830	Bettendorff & Wüllner.
,, (amorphous),	21 ,, 65	·0758	,, ,,
Bismuth,	9 ,, 102	·0298	Bède.
Boron (crystalline),	0 ,, 100	·2518	Mixter & Dana
,, (amorphous),	18 ,, 48	·254	Kopp.
Bromine, solid,	−78 ,, −20	·0843	Regnault.
,, liquid,	13 ,, 45	·1071	Andrews.
Cadmium,	0 ,, 100	·0548	Bunsen.
Calcium,	0 ,, 100	·1804	,,
Carbon, diamond,	11	·1128	H. F. Weber.
,, graphite,	11	·1604	,,
,, wood charcoal,	0 to 99	·1935	,,
Cobalt,	9 ,, 97	·1067	Regnault.
Copper,	15 ,, 100	·0933	Bède.
Gold,	0 ,, 100	·0316	Violle.
Iodine,	9 ,, 98	·0541	Regnault.
Iridium,	0 ,, 100	·0323	Violle.
Iron,	50	·1124	Byström.
Lead,	19 to 48	·0315	Kopp.
Lithium,	27 ,, 99	·9408	Regnault.
Magnesium,	20 ,, 51	·245	Kopp.
Manganese,	14 ,, 97	·1217	Regnault.
Mercury, solid,	−78 ,, −40	·0319	,,
,, liquid,	17 ,, 48	·0335	Kopp.
Molybdenum,	5 ,, 15	·0659	De la Rive and Marcet.
Nickel,	14 ,, 97	·1092	Regnault.
Osmium,	19 ,, 98	·0311	,,
Palladium,	0 ,, 100	·0592	Violle.
Phosphorus (yellow, solid)	−78 ,, 10	·1699	Regnault.
,, (,, liquid)	49 ,, 98	·2045	Person.
,, (red),	15 ,, 98	·1698	Regnault.
Platinum,	0 ,, 100	·0323	Violle.
Potassium,	−78 ,, 0	·1655	Regnault.

Substance.	Temperature.	Sp. Heat.	Observer.
Rhodium,	10 ,, 97	·0580	Regnault.
Selenium, crystalline,	22 ,, 62	·0840	Bettendorff & Wüllner.
Silicon, crystalline,	22	·1697	H. F. Weber.
Silver,	0 to 100	·0559	Bunsen.
Sodium,	−28 ,, 6	·2934	Regnault.
Sulphur (rhomb. cryst.),	17 ,, 45	·163	Kopp.
,, (newly melted),	15 ,, 97	·1844	Regnault.
Tellurium, crystalline,	21 ,, 51	·0475	Kopp.
Thallium,	17 ,, 100	·0335	Regnault.
Tin, cast,	0 ,, 100	·0559	Bunsen.
Zinc,	0 ,, 100	·0935	,,
Substances not Elementary.			
Brass (4 copper 1 tin), hard,	15° to 98°	·0858	Regnault.
,, ,, soft,	14 ,, 98	·0862	,,
Ice,	−20 ,, 0	·504	,,

107. The following determinations of specific heat of liquids are by Regnault. We have omitted decimal figures after the fourth, as even the second figure is different with different observers :—

Alcohol.		Chloroform.		Oil of Turpentine.	
Temp.	Sp. Ht.	Temp.	Sp. Ht.	Temp.	Sp. Ht.
−20°	·5053	−30°	·2293	−20°	·3842
0	·5475	0	·2324	0	·4106
40	·6479	30	·2354	40	·4538
80	·7694	60	·2384	80	·4842
				120	·5019
				160	·5068

Ether.		Bisulphide of Carbon.	
Temp.	Sp. Ht.	Temp.	Sp. Ht.
−30°	·5113	−30°	·2303
0	·5290	0	·2352
30	·5468	30	·2401

Schüller has found the specific heat of liquid benzine at $t°$ to be

$$·37980 + ·00144t.$$

108. The following table (from Miller's "Chemical Physics," p. 308) contains the results of Regnault's experiments on the specific heat of gases. The column headed "equal weights" contains the *specific heats* in the sense in which we have defined that term. The column headed "equal volumes" gives the relative thermal capacities of equal volumes at the same pressure and temperature :—

Thermal Capacities of Gases and Vapours at Constant Pressure.

Gas or Vapour.	Equal		Gas or Vapour.	Equal	
	Vols.	Weights.		Vols.	Weights.
Air,	·2375	·2375	Hydrochloric Acid,	·2352	·1842
Oxygen,	·2405	·2175	Sulphuretted Hydrogen,	·2857	·2432
Nitrogen,	·2368	·2438	Steam,	·2989	·4805
Hydrogen,	·2359	3·4090	Alcohol,	·7171	·4534
Chlorine,	·2964	·1210	Wood Spirit,	·5063	·4580
Bromine,	·3040	·0555	Ether,	1·2266	·4796
Nitrous Oxide,	·3447	·2262	Ethyl Chloride,	·6096	·2738
Nitric Oxide,	·2406	·2317	,, Bromide,	·7026	·1896
Carbonic Oxide	·2370	·2450	,, Disulphide,	1·2466	·4008
Carbonic Anhydride,	·3307	·2163	,, Cyanide,	·8293	·4261
Carbonic Disulphide,	·4122	·1569	Chloroform,	·6461	·1566
Ammonia,	·2996	·5084	Dutch Liquid,	·7911	·2293
Marsh Gas,	·3277	·5929	Acetic Ether,	1·2184	·4008
Olefiant Gas,	·4106	·4040	Benzol,	1·0114	·3754
Arsenious Chloride,	·7013	·1122	Acetone,	·8341	·4125
Silicic Chloride	·7778	·1322	Oil of Turpentine,	2·3776	·5061
Titanic ,,	·8564	·1290	Phosphorus Chloride,	·6386	·1347
Stannic ,,	·8639	·0939			
Sulphurous Anhydride,	·341	·154			

109. E. Wiedemann ("Pogg. Ann.," 1876, No. 1, p. 39) has made the following determinations of the specific heats of gases :—

Specific Heat.

	At 0°.	At 100°.	At 200°.	Relative Density.
Air,	0·2389	...	...	1
Hydrogen,	3·410	...	...	0·0692
Carbonic Oxide,	0·2426	...	...	0·967
Carbonic Acid,	0·1952	0·2169	0·2387	1·529
Ethyl,	0·3364	0·4189	0·5015	0·9677
Nitric Oxide,	0·1983	0·2212	0·2442	1·5241
Ammonia,	0·5009	0·5317	0·5629	0·5894

Multiplying the specific heat by the relative density, he obtains the following values of

Thermal Capacity of Equal Volumes.

	At 0°.	At 100°.	At 200°.
Air,	0·2389	...	...
Hydrogen,	0·2359	...	...
Carbonic Oxide,	0·2346	...	...
Carbonic Acid,	0·2985	0·3316	0·3650
Ethyl,	0·3254	0·4052	0·4851
Nitric Oxide,	0·3014	0·3362	0·3712
Ammonia,	0·2952	0·3134	0·3318

The same author ("Pogg. Ann.," 1877, New Series, vol. ii. p. 195) has made the following determinations of specific heats of vapours at temperature $t°$:—

Vapour.	Range of Temp. in Experiments.	Specific Heat.
Chloroform,	26°·9 to 189°·8	$·1341 + ·0001354t$
Bromic Ethyl,	27°·9 to 189°·5	$·1354 + ·003560t$
Benzine,	34°·1 to 115°·1	$·2237 + ·0010228t$
Acetone,	26°·2 to 179°·3	$·2984 + ·0007738t$
Acetic Ether,	32°·9 to 113°·4	$·2738 + ·0008700t$
Ether,	25°·4 to 188°·8	$·3725 + ·0008536t$

Regnault's determinations for the same vapours were as follows :—

Vapour.	Range of Temperature.	Mean Specific Heat for this Range. According to Regnault.	According to Wiedemann.
Chloroform, ...	117° to 228°	·1567	·1573
Bromic Ethyl, ...	77°·7 to 196°·5	·1896	·1841
Benzine, ...	116° to 218°	·3754	·3946
Acetone, ...	129° to 233°	·4125	·3946
Acetic Ether, ...	115° to 219°	·4008	·4190
Ether, ...	70° to 220°	·4797	·4943

Regnault has also determined the mean specific heat of bisulphide of carbon vapour between 80° and 147° to be ·1534, and between 80° and 229° to be ·1613.

Melting Points and Heat of Liquefaction.

110. Violle has made the following determinations of melting points ("Com. Ren.," lxxxix. p. 702) :—

Silver, ...	954°	Palladium, ...	1500°
Gold, ...	1045	Platinum, ...	1775
Copper, ...	1054	Iridium, ...	1950

This last temperature 1950° is very near to that of the hottest part of the oxyhydrogen flame.

The same observer has found the latent heat of liquefaction of platinum to be 27·2, and of palladium 36·3 ("Com. Ren." lxxxv. p. 543, and lxxxvii. p. 981).

111. The following approximate table of melting points is based on that given in the second supplement to Watt's "Dictionary of Chemistry," pp. 242, 243 :—

Platinum, ...	2000°	Copper, ...	1090°
Palladium, ...	1950	Silver, ...	1000
Gold, ...	1200	Borax, ...	1000
Cast Iron, ...	1200	Antimony, ...	432
Glass, ...	1100	Zinc, ...	360

Lead,	330°	Wax,	68°
Cadmium,	320	Potassium,	58
Bismuth,	265	Paraffin,	54
Tin,	230	Spermaceti,	44
Selenium,	217	Phosphorus,	43
Cane Sugar,	160	Water,	0
Sulphur,	111	Bromine,	−21
Sodium,	90	Mercury,	−40

112. The following table (from Watt's "Dictionary of Chemistry," vol. iii. p. 77) exhibits the latent heats of fluidity of certain substances, together with their melting points :—

	Melting Point.	Latent Heat.		Melting Point.	Latent Heat.
Mercury,	−39°	2·82	Silver,	1000°	21·1
Phosphorus,	44	5·0	Zinc,	433	28·1
Lead,	332	5·4	Chloride of Calcium ($CaCl.3H^2O$),	28·5	40·7
Sulphur,	115	9·4	Nitrate of Potassium,	339	47·4
Iodine,	107	11·7	Nitrate of Sodium,	310·5	63·0
Bismuth,	270	12·6			
Cadmium,	320	13·6			
Tin,	235	14·25			

The latent heat of fluidity of water was found by Regnault, and by Provostaye and Desains, to be 79°. Bunsen, by means of his ice-calorimeter ("Pogg. Ann.," vol. cxli. p. 30), has obtained the value 80·025. He finds the specific gravity of ice to be ·9167.

113. Chandler Roberts and Wrightson have compared the densities of molten and solid metals by weighing a solid metal ball in a bath of molten metal either of the same or a different kind ("Phys. Soc.," 1881, p. 195, and 1882, p. 102). They find that "iron expands rapidly (as much as 6 per cent.) in cooling from the liquid to the plastic state, and then contracts 7 per cent. to solidity; whereas bismuth appears to expand in cooling from the liquid to the solid state about 2·35 per cent." The following is their tabular statement of results :—

Metal.	Sp. Grav. of Solid.	Sp. Grav. of Liquid.	Percentage of change in volume from cold solid to liquid.
Bismuth,	9·82	10·055	Decrease of vol. 2·3
Copper,	8·8	8·217	Increase of vol. 7·1
Lead,	11·4	10·37	,, 9·93
Tin,	7·5	7·025	,, 6·76
Zinc,	7·2	6·48	,, 11·1
Silver,	10·57	9·51	,, 11·2
Iron (No. 4 foundry, Cleveland),	6·95	6·88	,, 1·02

114. Change of volume in melting, from Kopp's experiments (Watt's "Dic.," Art. *Heat*, p. 78):—

Phosphorus. Calling the volume at 0° unity, the volume at the melting point (44°) is 1·017 in the solid, and 1·052 in the liquid, state.

Sulphur. Volume at 0° being 1, volume at the melting point (115°) is 1·096 in the solid, and 1·150 in the liquid, state.

Wax. Volume at 0° being 1, volume at melting point (64°) is 1·161 in solid, and 1·166 in liquid, state.

Stearic Acid. Volume at 0° being 1, volume at melting point (70°) is 1·079 in solid, and 1·198 in liquid, state.

Rose's Fusible Metal (2 parts bismuth, 1 tin, 1 lead). Volume at 0° being 1, volume at 59° is a maximum, and is 1·0027. Volume at melting point (between 95° and 98°) is greater in liquid than in solid state by 1·55 per cent.

115. The following table (from Miller's "Chemical Physics," p. 344) exhibits the change of volume of certain substances in passing from the liquid to the vaporous condition under the pressure of one atmosphere:—

1 volume of	water yields	1696	volumes of	vapour.
,,	alcohol	528	,,	,,
,,	ether	298	,,	,,
,,	oil of turpentine	193	,,	,,

116. The following table of boiling points and heats of vaporization, at atmospheric pressure, is condensed from Landolt and Börnstein, pp. 189, 190 :—

	Boiling Point.	Latent Heat of Vaporization.	Observer.
Alcohol,	77·9	202·4	Andrews.
Bisulphide of Carbon,	46·2	86·7	,,
Bromine,	58	45·6	,,
Ether,	34·9	90·4	,,
Mercury,	350	62·0	Person.
Oil of Turpentine,	159·3	74·0	Brix.
Sulphur,	316	362·0	Person.
Water,	100	535·9	Andrews.

117. Regnault's approximate formula for what he calls "the total heat of steam at $t°$," that is, for the heat required to raise unit mass of water from 0° to $t°$ in the liquid state and then convert it into steam at $t°$, is

$$606{\cdot}5 + {\cdot}305t.$$

If the specific heat of water were the same at all temperatures, this would give

$$606{\cdot}5 - {\cdot}695t$$

as the heat of evaporation at $t°$. But since, according to Regnault, the heat required to raise the water from 0° to $t°$ is

$$t + {\cdot}000\ 02t^2 + {\cdot}000\ 000\ 3t^3,$$

the heat of evaporation will be the difference between this and the "total heat," that is, will be

$$606{\cdot}5 - {\cdot}695t - {\cdot}000\ 02t^2 - {\cdot}000\ 000\ 3t^3,$$

which is accordingly the value adopted by Regnault as the heat of evaporation of water at $t°$.

118. According to Regnault, the increase of pressure at constant volume, and increase of volume at constant

pressure, when the temperature increases from 0° to 100°, have the following values for the gases named :—

Gas.	At Constant Volume.	At Constant Pressure.
Hydrogen.....	·3667	·3661
Air,.....	·3665	·3670
Nitrogen, ...	·3668	...
Carbonic Oxide,.....	·3667	·3669
Carbonic Acid,.....	·3688	·3710
Nitrous Oxide,...	·3676	·3719
Sulphurous Acid,.....	·3845	·3903
Cyanogen, ...	·3829	·3877

Jolly has obtained the following values for the coefficient of increase of pressure at constant volume :—

Air, ...	·00366957
Oxygen,.....	·00367430
Hydrogen, ...	·00365620
Nitrogen,..... ..	·0036677
Carbonic Acid,.....	·0037060
Nitrous Oxide,.....	·0037067

Mendelejeff and Kaiander have determined the coefficient of expansion of air at constant pressure to be ·0036843.

119. Regnault's results as to the departures from Boyle's law are given in the form—

$$\frac{V_1 P_1}{V_0 P_0} = 1 \pm A(m-1) \pm B(m-1)^2,$$

V_1 denoting the volume at the pressure P_1, V_0 the volume at atmospheric pressure P_0, and m the ratio $\frac{V_0}{V_1}$.

For air, the negative sign is prefixed to A and the positive sign to B, and we have

$$\log A = \bar{3}\cdot 0435120,$$
$$\log B = \bar{5}\cdot 2873751.$$

For nitrogen, the signs are the same as for air, and we have

$$\log A = \bar{4}\cdot 8399375,$$
$$\log B = \bar{6}\cdot 8476020.$$

For carbonic acid, the negative sign is to be prefixed both to A and B, and we have

$$\log A = \bar{3}\cdot 9310399,$$
$$\log B = \bar{6}\cdot 8624721.$$

For hydrogen, the positive sign is to be prefixed both to A and B, and we have

$$\log A = \bar{4}\cdot 7381736,$$
$$\log B = \bar{6}\cdot 9250787.$$

120. The following table, showing the maximum pressure of aqueous vapour at temperatures near the ordinary boiling point, is based on Regnault's determinations, as revised by Moritz (Guyot's Tables, second edition, collection D, table xxv.):—

Temperature.	Centims. of Mercury at Paris.	Dynes per sq. cm.
99·0°	73·319	$9\cdot779 \times 10^5$
99·1	73·584	9·814 ,,
99·2	73·849	9·849 ,,
99·3	74·115	9·885 ,,
99·4	74·382	9·920 ,,
99·5	74·650	9·956 ,,
99·6	74·918	9·992 ,,
99·7	75·187	$1\cdot0028 \times 10^6$
99·8	75·457	1·0064 ,,
99·9	75·728	1·0100 ,,
100·0	76·000	1·0136 ,,
100·1	76·273	1·0173 ,,
100·2	76·546	1·0209 ,,
100·3	76·820	1·0245 ,,

Temperature.	Centims. of Mercury at Paris.	Dynes per sq. cm.
100·4	77·095	$1·0282 \times 10^6$
100·5	77·371	1·0319 ,,
100·6	77·647	1·0356 ,,
100·7	77·925	1·0393 ,,
100·8	78·203	1·0430 ,,
100·9	78·482	1·0467 ,,
101·0	78·762	1·0505 ,,

121. *Maximum Pressure of Aqueous Vapour at various temperatures, in dynes per sq. centim.*

−20°	1236	50°	$1·226 \times 10^5$
−15	1866	60	1·985 ,,
−10	2790	80	4·729 ,,
− 5	4150	100	$1·014 \times 10^6$
0	6133	120	1·988 ,,
5	8710	140	3·626 ,,
10	12220	160	6·210 ,,
15	16930	180	$1·006 \times 10^7$
20	23190	200	1·560
25	31400		
30	42050		
40	73200		

Maximum Pressure of various Vapours, in dynes per sq. cm.

	Alcohol.	Ether.	Sulphide of Carbon.	Chloroform.
−20°	4455	$9·19 \times 10^4$	$6·31 \times 10^4$	
−10	8630	$1·53 \times 10^5$	$1·058 \times 10^5$	
0	16940	2·46 ,,	1·706 ,,	
10	32310	3·826 ,,	2·648 ,,	
20	59310	5·772 ,,	3·975 ,,	$2·141 \times 10^5$
30	$1·048 \times 10^5$	8·468 ,,	5·799 ,,	3·301 ,,
40	1·783 ,,	$1·210 \times 10^6$	8·240 ,,	4·927 ,,
50	2·932 ,,	1·687 ,,	$1·144 \times 10^6$	7·14 ,,
60	4·671 ,,	2·301 ,,	1·554 ,,	$1·007 \times 10^6$
80	$1·084 \times 10^6$	4·031 ,,	2·711 ,,	1·878 ,,
100	2·265 ,,	6·608 ,,	4·435 ,,	3·24 ,,
120	4·31 ,,	$1·029 \times 10^7$	6·87 ,,	5·24 ,,

122. The following are approximate values of the maximum pressure of aqueous vapour at various temperatures, in millimetres of mercury. They can be reduced to dynes per sq. cm. by multiplying by 133·4 :—

	mm.		mm.		mm.		mm.
0°	4·6	92°	567	112°	1150	132°	2155
10	9·2	94	611	114	1228	134	2286
20	17·4	96	658	116	1311	136	2423
30	31·5	98	707	118	1399	138	2567
40	54·9	100	760	120	1491	140	2718
50	96·2	102	816	122	1588	142	2875
60	149	104	875	124	1691	144	3040
70	233	106	938	126	1798	146	3213
80	355	108	1004	128	1911	148	3393
90	525	110	1075	130	2030	150	3581

123. The density (in gm. per cub. cm.) of aqueous vapour at any temperature t and any pressure p (dynesper sq. cm.), whether equal to or less than the maximum pressure, is

$$\frac{\cdot 622 \times \cdot 001276}{1 + \cdot 00366t} \times \frac{p}{10^6}.$$

If q denote the pressure in millimetres of mercury, the approximate formula is

$$\frac{\cdot 622 \times \cdot 001293}{1 \times \cdot 00366t} \times \frac{q}{760}.$$

124. Temperature of evaporation and dew-point (Glaisher's Tables, second edition, page iv.). The following are the factors by which it is necessary to multiply the excess of the reading of the dry thermometer over that of the wet, to give the excess of the temperature of the air above that of the dew-point :—

Reading of Dry Bulb Therm.		Factor.	Reading of Dry Bulb Therm.		Factor.
−10° C.	= 14° F.	8·76	15° C.	= 59° F.	1·89
− 5	23	7·28	20	68	1·79
0	32	3·32	25	77	1·70
+ 5	41	2·26	30	86	1·65
+10	50	2·06	35	95	1·60

125. Critical temperatures of gases, above which they cannot be liquefied (abridged from Landolt and Börnstein, p. 62):—

	Critical Temperature. °	Max. Pressure of Gas at this Temp.	Observer.
Hydrogen,	−174·2	98·9 atm.	Sarrau.
Oxygen,	−105·4	48·7 ,,	,,
Nitrogen,	−123·8	42·1 ,,	,,
Carbonic Acid,	30·92	,,	Andrews.
,,	32·0	77·0 ,,	Sarrau.
Bisulphide of Carbon,	271·8	74·7 ,,	Sajotschewsky.
Sulphurous Acid,	155·4	78·9 ,,	,,
Chloroform,	260·0	54·9 ,,	,,
Benzol,	280·6	49·5 ,,	,,
Alcohol,	234·3	62·1 ,,	,,
Ether,	190·0	36·9 ,,	,,

Conductivity.

126. By the *thermal conductivity* of a substance *at a given temperature* is meant the value of k in the expression

$$Q = kA\frac{v_2 - v_1}{x}t, \quad \ldots \ldots \quad (1)$$

where Q denotes the quantity of heat that flows, in time t, through a plate of the substance of thickness x, the area of each of the two opposite faces of the plate being A, and the temperatures of these faces being respectively v_1 and v_2, each differing but little from the given temperature. The lines of flow of heat are supposed to be normal to the faces, or, in other words, the isothermal surfaces within the plate are supposed to be parallel to the faces; and the flow of heat is supposed to be *steady*, in other words, no part of the plate is to be gaining or losing heat on the whole.

Briefly, and subject to these understandings, conductivity may be defined as *the quantity of heat that passes in unit time, through unit area of a plate whose thickness is unity, when its opposite faces differ in temperature by one degree.*

127. *Dimensions of Conductivity.* From equation (1) we have

$$k = \frac{Q}{v_2 - v_1} \cdot \frac{x}{At} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The dimensions of the factor $\frac{Q}{v_2 - v_1}$ are simply M, since the unit of heat varies jointly as the unit of mass and the length of the degree. The dimensions of the factor $\frac{x}{At}$ are $\frac{1}{LT}$; hence the dimensions of k are $\frac{M}{LT}$. This is on the supposition that the unit of heat is the heat required to raise *unit mass* of water one degree. In calculations relating to conductivity it is perhaps more usual to adopt as the unit of heat the heat required to raise *unit volume* of water one degree. The dimensions of $\frac{Q}{v_2 - v}$ will then be L^3, and the dimensions of k will be $\frac{L^2}{T}$.

These conclusions may be otherwise expressed by saying that the dimensions of conductivity are $\frac{M}{LT}$ when the thermal capacity of unit mass of water is taken as unity, and are $\frac{L^2}{T}$ when the capacity of unit volume of water is taken as unity. In the C.G.S. system the capacities of unit mass and unit volume of water are practically identical.

128. Let c denote the thermal capacity of unit volume of a substance through which heat is being conducted. Then $\frac{k}{c}$ denotes a quantity whose value it is often necessary to discuss in investigations relating to the transmission of heat. We have, from equation (2),

$$\frac{k}{c} = \frac{Q'}{v_2 - v_1} \cdot \frac{x}{At},$$

where Q′ denotes $\frac{Q}{c}$. Hence $\frac{k}{c}$ would be the numerical value of the conductivity of the substance, if the unit of heat employed were the heat required to raise unit volume of the substance one degree. Professor Clerk Maxwell proposed to call $\frac{k}{c}$ the *thermometric* conductivity, as distinguished from k the *thermal* or *calorimetric* conductivity. We prefer, in accordance with Sir Wm. Thomson's article, "Heat," in the *Encyclopædia Britannica*, to call $\frac{k}{c}$ the *diffusivity* of the substance for heat, a name which is based on the analogy of $\frac{k}{c}$ to a coefficient of diffusion.

Coefficient of Diffusion.

129. There is a close analogy between conduction and diffusion. Let x denote the distance between two parallel plane sections A and B to which the diffusion is perpendicular, and let these sections be maintained in constant states. Then, if we suppose one substance to be at rest, and another substance to be diffusing through it, the coefficient of diffusion K is defined by the equation

$$y = \underset{x}{K}^{t}, \quad \ldots \ldots \quad (1)$$

where y denotes the thickness of a stratum of the mixture as it exists at B, which would be reduced to the state existing at A by the addition to it of the quantity which diffuses from A to B in the time t.

When the thing diffused is heat, the states at A and B are the temperatures v_1 and v_2, and y denotes the thickness of a stratum at the lower temperature which would be raised to the higher by the addition of as much heat as passes in the time t. This quantity of heat, for unit area, will be

$$\frac{kt}{x}(v_2 - v_1),$$

which must therefore be equal to

$$yc(v_2 - v_1),$$

whence we have

$$y = \frac{k}{c}\,\frac{t}{x}.$$

The "thermometric conductivity" $\frac{k}{c}$ may therefore be regarded as the coefficient of diffusion of heat.

130. When we are dealing with the mutual interdiffusion of two liquids, or of two gases contained in a closed vessel, subject in both cases to the law that the volume of a mixture of the two substances is the sum of the volumes of its components at the same pressure, the quantity of one of the substances which passes any section in one direction must be equal (in volume) to the quantity of the other which passes it in the opposite direction, since the total volume on either side of the section remains unaltered; and a similar equality must hold for the quantities which pass across the interval between two sections, provided that the absorption in the interval

itself is negligible. Let x as before denote the distance between two parallel plane sections A and B to which the diffusion is perpendicular. Let the mixture at A consist of m parts by volume of the first substance to $1-m$ of the second, and the mixture at B consist of n parts of the second to $1-n$ of the first, m being greater than $1-n$, and therefore n greater than $1-m$. The first substance will then diffuse from A to B, and the second in equal quantity from B to A. Let each of these quantities be such as would form a stratum of thickness z (the vessel being supposed prismatic or cylindrical, and the sections considered being normal sections), then z will be proportional to

$$\frac{m-(1-n)}{x}t, \quad \text{that is to} \quad \frac{m+n-1}{x}t,$$

and the coefficient of interdiffusion K is defined by the equation

$$z = \mathrm{K}\,\frac{m+n-1}{x}t. \qquad (2)$$

The numerical quantity $m+n-1$ may be regarded as measuring the difference of states of the two sections A and B.

If y now denote the thickness of a stratum in the condition of B which would be reduced to the state existing at A by the abstraction of a thickness z of the second substance, and the addition of the same thickness of the first, we have $(1-n)y+z$ as the expression for the quantity of the first substance in the stratum after the operation. This is to be equal to my. Hence we have

$$y = \frac{z}{m+n-1}, \qquad (3)$$

and substituting for z its value in (2) we have finally

$$y = K\frac{t}{x}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is of the same form as equation (1), y now denoting the thickness of a stratum of the mixture as it exists at B, which would be reduced to the state existing at A by the addition to it of the quantity of one substance which diffuses from A to B in the time t, and the removal from it of the quantity of the other substance which diffuses from B to A in the same time.

131. The following values of K in terms of the centimetre and second are given in Professor Clerk Maxwell's "Theory of Heat," 4th edition, p. 332, on the authority of Professor Loschmidt of Vienna.

Coefficients of Interdiffusion of Gases.

Carbonic Acid and Air,	·1423
,, ,, Hydrogen,	·5614
,, ,, Oxygen,	·1409
,, ,, Marsh Gas,	·1586
,, ,, Carbonic Oxide,	·1406
,, ,, Nitrous Oxide,	·0982
Oxygen and Hydrogen,	·7214
,, ,, Carbonic Oxide,	·1802
Carbonic Oxide and Hydrogen,	·6422
Sulphurous Acide and Hydrogen,	·4800

These may be compared with the value of $\frac{k}{c}$ for air, which, according to Professor J. Stefan of Vienna, is ·256.

The value of k for air, according to the same authority, is $5{\cdot}58 \times 10^{-5}$, and is independent of the pressure. Professor Maxwell, by a different method, calculates its value at $5{\cdot}4 \times 10^{-5}$.

Results of Experiments on Conductivity of Solids.

132. Principal Forbes' results for the conductivity of iron (Stewart on Heat, p. 261, second edition) are expressed in terms of the foot and minute, the cubic foot of water being the unit of thermal capacity. Hence the value of Forbes' unit of conductivity, when referred to C.G.S., is $\frac{(30{\cdot}48)^2}{60}$, or 15·48; and his results must be multiplied by 15·48 to reduce them to the C.G.S. scale. His observations were made on two square bars; the side of the one being $1\frac{1}{4}$ inch, and of the other an inch. The results when reduced to C.G.S. units are as follows:—

Temp. Cent.	$1\frac{1}{4}$-inch bar.	1-inch bar.
0°	·207	·1536
25	·1912	·1460
50	·1771	·1399
75	·1656	·1339
100	·1567	·1293
125	·1496	·1259
150	·1446	·1231
175	·1399	·1206
200	·1356	·1183
225	·1317	·1160
250	·1279	·1140
275	·1240	·1121

133. Neumann's results ("Ann. de. Chim." vol. lxvi. p. 185) must be multiplied by ·000848 to reduce them to our scale. They then become as follows:—

Copper,	1·108
Brass,	·302
Zinc,	·307
Iron,	·164
German Silver,	·109
Ice,	·0057

In the same paper he gives for the following substances the values of $\frac{k}{sd}$ or $\frac{k}{c}$; that is, the quotient of conductivity by the thermal capacity of unit volume. These require the same reducing factor as the values of k, and when reduced to our scale are as follows :—

	Values of $\frac{k}{c}$.
Coal,	·00116
Melted Sulphur,	·00142
Ice,	·0114
Snow,	·00356
Frozen Mould,	·00916
Sandy Loam,	·0136
Granite (coarse),	·0109
Serpentine,	·00594

134. Sir W. Thomson's results, deduced from observations of underground thermometers at three stations at Edinburgh ("Trans. R. S. E.," 1860, p. 426), are given in terms of the foot and second, the thermal capacity of a cubic foot of water being unity, and must be multiplied by $(30{\cdot}48)^2$ or 929 to reduce them to our scale. The following are the reduced results :—

	k, or Conductivity.	$\frac{k}{c}$.
Trap-rock of Calton Hill,	·00415	·00786
Sand of experimental garden,	·00262	·00872
Sandstone of Craigleith Quarry,	·01068	·02311

My own result for the value of $\frac{k}{c}$ from the Greenwich underground thermometers ("Greenwich Observations," 1860) is in terms of the French foot and the year. As a French foot is 32·5 centims., and a year is 31557000 seconds the reducing factor is $(32{\cdot}5)^2 \div 31557000$; that is, $3{\cdot}347 \times 10^{-5}$. The result is

	$\frac{k}{c}$.
Gravel of Greenwich Observatory Hill,	·01249

Professors Ayrton and Perry ("Phil. Mag.," April, 1878) determined the conductivity of a Japanese building stone (porphyritic trachyte) to be ·0059.

135. Ångström, in "Pogg. Ann.," vols. cxiv. (1861) and cxviii. (1863), employs as units the centimetre and the minute; hence his results must be divided by 60. These results, as given at p. 294 of his second paper, will then stand as follows:—

	Value of $\frac{k}{c}$.
Copper, first specimen, ...	$1{\cdot}216\,(1 - {\cdot}00214\,t)$
,, second specimen, ...	$1{\cdot}163\,(1 - {\cdot}001519\,t)$
Iron,..... ..	$\cdot224\,(1 - {\cdot}002874\,t)$

He adopts for c the values

·84476 for copper; ·88620 for iron,

and thus deduces the following values of k:—

	Conductivity.
Copper, first specimen, ...	$1{\cdot}027\,(1 - {\cdot}00214\,t)$
,, second specimen, ...	$\cdot983\,(1 - {\cdot}001519\,t)$
Iron,.....	$\cdot199\,(1 - {\cdot}002874\,t)$

136. A Committee, consisting of Professors Herschel and Lebour, and Mr. J. F. Dunn, appointed by the British Association to determine the thermal conductivities of certain rocks, have obtained results from which the following selection has been made by Professor Herschel:—

Substance.	Conductivity in C.G.S. Units.	$\frac{k}{c}$
Iron pyrites, more than	·01 more than	·0170
Rock salt, rough crystal,.....	·0113	·0288
Fluorspar, rough crystal, ...	·00963	·0156
Quartz, opaque crystal, and quartzites,.....	·0080 to ·0092	·0175 to ·0190
Siliciousandstones(slightlywet),	·00641 to ·00854	·0130 to ·0230

Substance.	Conductivity in C.G.S. Units.	$\frac{k}{c}$
Galena, rough crystal, interspersed with quartz,	·00705	·0171
Sandstone and hard grit, dry,	·00545 to ·00565	·0120
Sandstone and hard grit, thoroughly wet,	·00590 to ·00610	·0100
Micaceous flagstone, along the cleavage,	·00632	·0116
Micaceous flagstone, across cleavage,	·00441	·0087
Slate, along cleavage,	·00550 to ·00650	·0102
Do., across cleavage,	·00315 to ·00360	·0057
Granite, various specimens, about	·00510 to ·00550	·0100 to ·0120
Marbles, limestone, calcite, and compact dolomite,	·00470 to ·00560	·0085 to ·0095
Red Serpentine (Cornwall),	·00441	·0065
Caen stone (building limestone),	·00433	·0089
Whinstone, trap rock, and mica schist,	·00280 to ·00480	·0055 to ·0095
Clay slate (Devonshire),	·00272	·0053
Tough clay (sun-dried),	·00223	·0048
Do., soft (with one-fourth of its weight of water),	.00310	·0035
Chalk,	·00200 to ·00330	·0046 to ·0059
Calcareous sandstone (firestone),	·00211	·0049
Plate-glass German and English,	·00198 to ·00234	·00395 to ?
German glass toughened,	·00185	·00395
Heavy spar, opaque rough crystal,	·00177	.....
Fire-brick,	·00174	·0053
Fine red brick,	·00147	·0044
Fine plaster of Paris, dry plate,	·00120	·0060 } about
Do., thoroughly wet,	·00160	·0025 }
White sand, dry,	·00093	·0032
Do., saturated with water, about	·00700	·0120 about
House coal and cannel coal,	·00057 to ·00113	·0012 to ·0027
Pumice stone,	·00055	.....

137. Péclet in "Annales de Chimie," sér. 4, tom. ii. p. 114 [1841], employs as the unit of conductivity the transmission, in one second, through a plate a metre square and a millimetre thick, of as much heat as will raise a cubic decimetre (strictly a kilogramme) of water one degree. Formula (2) shows that the value of this conductivity in the C.G.S. system, is

$$\frac{1000}{1}\ \frac{\frac{1}{10}}{10000};\ \text{that is,}\ \frac{1}{100}.$$

His results must accordingly be divided by 100.

The same author published in 1853 a greatly extended series of observations, in a work entitled "Nouveaux documents relatifs aux chauffage et à la ventilation." In this series the conductivity which is adopted as unity is the transmission, in one hour, through a plate a metre square and a metre thick, of as much heat as will raise a kilogramme of water one degree. This conductivity, in C.G.S. units is

$$\frac{1000}{1}\cdot\frac{100}{10000}\cdot\frac{1}{3600};\ \text{that is,}\ \frac{1}{360}.$$

The results must therefore be divided by 360. Those of them which refer to metals appear to be much too small. The following are for badly conducting substances :—

	Density.	Conductivity.
Fir, across fibres,	·48	·00026
,, along fibres,	·48	·00047
Walnut, across fibres,	...	·00029
,, along fibres,	...	·00048
Oak, across fibres,	...	·00059
Cork,	·22	·00029
Caoutchouc,	...	·00041
Gutta percha,	...	·00048
Starch paste,	1·017	·00118
Glass,	2·44	·0021

	Density.	Conductivity.
Glass,	2·55	·0024
Sand, quartz,	1·47	·00075
Brick, pounded, coarse-grained,	1·0	·00039
,, passed through silk sieve,	1·76	·00046
Fine brick dust, obtained by decantation,	1·55	·00039
Chalk, powdered, slightly damp,	·92	·00030
,, washed and dried,	·85	·00024
,, washed, dried, and compressed,	1·02	·00029
Potato-starch,	·71	·00027
Wood-ashes,	·45	·00018
Mahogany sawdust,	·31	·00018
Wood charcoal, ordinary, powdered,	·49	·00022
Bakers' breeze, in powder, passed through silk sieve,	·25	·00019
Ordinary wood charcoal in powder, passed through silk sieve,	·41	·000225
Coke, powdered,	·77	·00044
Iron filings,	2·05	·00044
Binoxide of manganese,	1·46	·00045
Woolly Substances.		
Cotton Wool, of all densities,	...	·000111
Cotton swansdown (molleton de coton), of all densities,	...	·000111
Calico, new, of all densities,	...	·000139
Wool, carded, of all densities,	...	·000122
Woollen swansdown (molleton de laine) of all densities,	...	·000067
Eider-down,	...	·000108
Hempen cloth, new,	·54	·000144
,, old,	·58	·000119
Writing-paper, white,	·85	·000119
Grey paper, unsized,	·48	·000094

138. In Professor George Forbes's paper on conductivity ("Proc. R. S. E.," February, 1873) the units are the centim. and the minute; hence his results must be divided by 60.

In a letter dated March 4, 1884, to the author of this work, Professor Forbes remarks that the mean temperature of the substances in these experiments was $-10°$, and expresses the opinion that bad conductors (such as most of these substances) conduct worse at low than at high temperatures—an opinion which was suggested by the analogy of electrical insulators. His results reduced to C.G.S. are—

Ice, along axis,	·00223	Kamptulikon,	·00011
Ice, perpendicular to axis,	·00213	Vulcanized india-rubber,	·000 089
Black marble,	·00177	Horn,	·000 087
White marble,	·00115	Beeswax,	·000 087
Slate,	·00081	Felt,	·000 087
Snow,	·00072	Vulcanite,	·000 0833
Cork,	·000717	Haircloth,	·000 0402
Glass,	·0005	Cotton-wool, divided,	·000 0433
Pasteboard,	·000453	,, pressed,	·000 0335
Carbon,	·000405	Flannel,	·000 0355
Roofing-felt,	·000335	Coarse linen,	·000 0298
Fir, parallel to fibre,	·0003	Quartz, along axis,	·000922
Fir, across fibre, and along radius,	·000 088	,,	·00124
Boiler-cement,	·000162	,,	·00057
Paraffin,	·00014	,,	·00083
Sand, very fine,	·000131	Quartz, perpendicular to axis,	·0040
Sawdust,	·000123	,,	·0044

Professor Forbes quotes a paper by M. Lucien De la Rive ("Soc. de Ph. et d'Hist. Nat. de Genève," 1864) in which the following result is obtained for ice,

Ice, ... ·00230.

M. De la Rive's experiments are described in "Annales de Chimie," sér. 4, tom. i. pp. 504-6.

139. Dr. Robert Weber ("Bulletin, Soc. Sciences Nat. de Neufchatel," 188), has found the following conductivities and surface emissivities for five specimens of rock from the St. Gothard tunnel :—

SPECIMEN No. 168.—*Micaceous Gneiss.*

Conductivity,.....	·000917 + ·000 0044t
Emissivity,.....	·000185 + ·000 0023t
Specific Heat,.....	·1778 + ·00042t

SPECIMEN No. 114.—*Mica Schist.*

Conductivity,.....	·000733 + ·000 010t
Emissivity,.....	·000207 + ·000 0016t
Specific Heat,.....	·18000 + ·00044t

SPECIMEN No. 124.—*Eurite.*

Conductivity,.....	·000862 + ·00016t
Emissivity,.....	·000249 + ·000 000 09t
Specific Heat,.....	·1682 + ·0006t

SPECIMEN No. 140.—*Gneiss.*

Conductivity,.....	·0014 + ·000 003t
Emissivity,.....	·00026 + ·000 0008t
Specific Heat,.....	·1463 + ·0009t

SPECIMEN No. 146.—*Micaceous Schist.*

Conductivity,.....	·000952 + ·000 009t
Emissivity,.....	·000168 + ·000 0023t
Specific Heat,.....	·1697 + ·0006t

Conductivity of Liquids.

140. The conductivity of water, according to experiments by Mr. J. T. Bottomley ("Phil. Trans." 1881, April 3), is ·002, which is nearly the same as the conductivity of ice. (See 138.)

141. H. F. Weber ("Sitz. kön. Preuss. Akad." 1885), has made the following determinations of conductivities of liquids at temperatures of from 9° to 15° C. He employs the centimetre, the gramme, and the minute as units: we have accordingly divided the original numbers by 60 to reduce to C.G.S.

	Conductivity.		Conductivity.
Water,	·00136	Amyl Acetate,	·000302
Aniline,	·000408		
Glycerine,	·000670	Chloro Benzol,	·000302
Ether,	·000303	Chloroform,	·000288
		Chloro Carbon,	·000252
Methyl Alcohol,	·000495	Propyl Chloride,	·000283
Ethyl Alcohol,	·000423	Isobutyl Chloride,	·000278
Propyl Alcohol,	·000373	Amyl Chloride,	·000284
Butyl Alcohol,	·000340		
Amyl Alcohol,	·000328	Bromo Benzol,	·000265
		Ethyl Bromide,	·000247
Ameisen Acid,	·000648	Propyl Bromide,	·000257
Acetic Acid,	·000472	Isobutyl Bromide,	·000278
Propion Acid,	·000390	Amyl Bromide,	·000237
Butyric Acid,	·000360		
Isobutyric Acid,	·000340	Ethyl Iodide,	·000222
Valerian Acid,	·000325	Propyl Iodide,	·000220
Isovalerian Acid,	·000312	Isobutyl Iodide,	·000208
Isocapron Acid,	·000298	Amyl Iodide,	·000203
Methyl Acetate,	·000385	Benzol,	·000333
Ethyl Formiate,	·000378	Toluol,	·000307
Ethyl Acetate,	·000348	Cymol,	·000272
Propyl Formiate,	·000357	Oil of Turpentine,	·000260
Propyl Acetate,	·000327		
Methyl Butyrate,	·000335	Sulphuric Acid,	·000765
Ethyl Butyrate,	·000318	Bisulphide of Carbon,	·000343
Methyl Valerate,	·000315	Oil of Mustard,	·000382
Ethyl Valerate,	·000307	Ethyl Sulphide,	·000328

In the original paper these numbers are compared with the thermal capacities of the liquids per unit of volume, and with the calculated mean distances between their molecules. It is found that conductivity, multiplied by mean distance, divided by capacity, is a nearly constant quantity for the members of any one of the above groups. Comparing one group with another, its most widely different values are represented by 19 and 23, if we except the last group, for which its value is between 26 and 27.

Emission and Surface Conduction.

142. Mr. D. M'Farlane has published ("Proc. Roy. Soc."

1872, p. 93) the results of experiments on the loss of heat from blackened and polished copper in air at atmospheric pressure. They need no reduction, the units employed being the centimetre, gramme, and second. The general result is expressed by the formulæ

$$x = \cdot000238 + 3\cdot06 \times 10^{-6}t - 2\cdot6 \times 10^{-8}t^2$$

for a blackened surface, and

$$x = \cdot000168 + 1\cdot98 \times 10^{-6}t - 1\cdot7 \times 10^{-8}t^2$$

for polished copper, x denoting the quantity of heat lost per second per square centim. of surface of the copper, per degree of difference between its temperature and that of the walls of the enclosure. These latter were blackened internally, and were kept at a nearly constant temperature of 14° C. The air within the enclosure was kept moist by a saucer of water. The greatest difference of temperature employed in the experiments (in other words, the highest value of t) was 50° or 60° C.

The following table contains the values of x calculated from the above formulæ, for every fifth degree, within the limits of the experiments :—

Difference of Temperature.	Value of x.		Ratio.
	Polished Surface.	Blackened Surface.	
5°	·000178	·000252	·707
10	·000186	·000266	·699
15	·000193	·000279	·692
20	·000201	·000289	·695
25	·000207	·000298	·694
30	·000212	·000306	·693
35	·000217	·000313	·693
40	·000220	·000319	·693
45	·000223	·000323	·690
50	·000225	·000326	·690
55	·000226	·000328	·690
60	·000226	·000328	·690

143. Professor Tait has published ("Proc. R. S. E." 1869-70, p. 207) observations by Mr. J. P. Nichol on the loss of heat from blackened and polished copper, in air, at three different pressures, the enclosure being blackened internally and surrounded by water at a temperature of approximately 8° C.* Professor Tait's units are the grain-degree for heat, the square inch for area, and the hour for time. The rate of loss per unit of area is

$$\frac{\text{heat emitted}}{\text{area} \times \text{time}}.$$

The grain-degree is ·0648 gramme-degree.

The square inch is 6·4516 square centims.

The hour is 3600 seconds.

Hence Professor Tait's unit rate of emission is

$$\frac{\cdot 0648}{6\cdot 4516 \times 3600} = 2\cdot 79 \times 10^{-6}$$

of our units. Employing this reducing factor, Professor Tait's Table of Results will stand as follows:—

Pressure $1\cdot 014 \times 10^6$ [760 millims. of mercury].

Blackened.		Bright.	
Temp. Cent.	Less per sq. cm. per second.	Temp. Cent.	Loss per sq. cm. per second.
61·2	·01746	63·8	·00987
50·2	·01360	57·1	·00862
41·6	·01078	50·5	·00736
34·4	·00860	44·8	·00628
27·3	·00640	40·5	·00562
20·5	·00455	34·2	·00438
		29·6	·00378
		23·3	·00278
		18·6	·00210

* This temperature is not stated in the "*Proceedings*," but has been communicated to me by Professor Tait.

Pressure $1{\cdot}36 \times 10^5$ [102 millims. of mercury].

62·5	·01298	67·8	·00492
57·5	·01158	61·1	·00433
53·2	·01048	55	·00383
47·5	·00898	49·7	·00340
43	·00791	44·9	·00302
28·5	·00490	40·8	·00268

Pressure $1{\cdot}33 \times 10^4$ [10 millims. of mercury].

62·5	·01182	65	·00388
57·5	·01074	60	·00355
54·2	·01003	50	·00286
41·7	·00726	40	·00219
37·5	·00639	30	·00157
34	·00569	23·5	·00124
27·5	·00446		
24·2	·00391		

Mechanical Equivalent of Heat.

144. The value originally deduced by Joule from his experiments on the stirring of water was 772 foot-pounds of work (at Manchester) for as much heat as raises a pound of water through 1° Fahr. This is 1389·6 foot-pounds for a pound of water raised 1° C., or 1389·6 foot-grammes for a gramme of water raised 1° C. As a foot is 30·48 centims., and the value of g at Manchester is 981·3, this is $1389{\cdot}6 \times 30{\cdot}48 \times 981{\cdot}3$ ergs per gramme-degree; that is, $4{\cdot}156 \times 10^7$ ergs per gramme-degree.

A later determination by Joule ("Brit. Assoc. Report," 1867, pt. i. p. 522, or "Reprint of Reports on Electrical Standards," p. 186) is 25187 foot-grain-second units of work per grain-degree Fahr. This is 45337 of the same units per grain-degree Centigrade, or 45337 foot-gramme-second units of work per gramme-degree Centigrade; that is to say,

$$45337 \times (30{\cdot}48)^2 = 4{\cdot}212 \times 10^7$$

ergs per gramme-degree Centigrade.

In view of the fact that the B.A. standard of electrical resistance employed in this determination is now known to be too small by about 1·3 per cent., and that the current energy converted into heat was accordingly underestimated to this extent, the result ought now to be increased by 1·3 per cent., which will make it

$$4{\cdot}267 \times 10^7.$$

At the meeting of the Royal Society, January, 1878 ("Proceedings," vol. xxvii. p. 38), an account was given by Joule of experiments recently made by him with a view to increase the accuracy of the results given in his former paper. ("Phil. Trans.," 1850.) His latest result from the thermal effects of the friction of water, as announced at this meeting, is, that taking the unit of heat as that which can raise a pound of water, weighed *in vacuo*, from 60° to 61° of the mercurial Fahrenheit thermometer; its mechanical equivalent, reduced to the sea-level at the latitude of Greenwich, is 772·55 foot-pounds.

To reduce this to water at 0° C. we have to multiply by 1·00089,* giving 773·24 ft. lbs., and to reduce to ergs per gramme-degree Centigrade we have to multiply by

$$981{\cdot}17 \times 30{\cdot}48 \times \frac{9}{5}.$$

The product is $4{\cdot}1624 \times 10^7$.

145. Some of the best determinations by various experimenters are given (in gravitation measure) in the following list, extracted from "Watts' Dictionary of Chemistry," Supplement 1872, p. 687. The value 429·3 in this list corresponds to $4{\cdot}214 \times 10^7$ ergs:—

* This factor is found by giving t the value 15·8 (since the temperature 60·5 Fahr. is 15·8 Cent.) in formula (3) of art. 101.

Hirn,.....	432	.....	Friction of water and brass.
,, ...	433	.....	Escape of water under pressure.
,, ...	441·6	.....	Specific heats of air.
,, ...	425·2	.....	Crushing of lead.
Joule,....	429·3	.....	Heat produced by an electric current.
Violle,...	435·2 (copper)..		Heat produced by induced currents.
	434·9 (aluminium)		
	435·8 (tin).....		
	437·4 (lead).....		
Regnault,	437	.....	Velocity of sound.

$J = e\left(\frac{p}{T} - \frac{1}{v}\right)$

We shall adopt $4{\cdot}2 \times 10^7$ ergs as the equivalent of 1 gramme-degree; that is, employing J as usual to denote Joule's equivalent, we have

$$J = 4{\cdot}2 \times 10^7 = 42 \text{ millions.}$$

146. *Heat and Energy of Combination with Oxygen.*

1 gramme of	Compound formed.	Gramme-degrees of heat produced.	Equivalent Energy, in ergs.
Hydrogen, ...	H^2O	34000 A F	$1{\cdot}43 \times 10^{12}$
Carbon, ...	CO^2	8000 A F	$3{\cdot}36 \times 10^{11}$
Sulphur, ...	SO^2	2300 A F	$9{\cdot}66 \times 10^{10}$
Phosphorus, ...	P^2O^5	5747 A	$2{\cdot}41 \times 10^{11}$
Zinc, ...	ZnO	1301 A	$5{\cdot}46 \times 10^{10}$
Iron, ...	Fe^3O^4	1576 A	$6{\cdot}62 \times 10^{10}$
Tin, ...	SnO^2	1233 A	5·18 ,,
Copper, ...	CuO	602 A	2·53 ,,
Carbonic oxide,....	CO^2	2420 A	$1{\cdot}02 \times 10^{11}$
Marsh-gas, ...	CO^2 and H^2O	13100 A F	5·50 ,,
Olefiant gas, ...	,,	11900 A F	5·00 ,,
Alcohol, ...	,,	6900 A F	2·90 ,,

Combustion in Chlorine.

Hydrogen, ...	HCl	23000 F T	$9{\cdot}66 \times 10^{11}$
Potassium, ...	KCl	2655 A	1·12 ,,
Zinc, ...	$ZnCl^2$	1529 A	$6{\cdot}42 \times 10^{10}$
Iron, ...	Fe^2Cl^6	1745 A	7·33 ,,
Tin, ...	$SnCl^4$	1079 A	4·53 ,,
Copper, ...	$CuCl^2$	961 A	4·04 ,,

The numbers in the last column are the products of the numbers in the preceding column by 42 millions.

The authorities for these determinations are indicated by the initial letters A (Andrews), F (Favre and Silbermann), T (Thomsen). Where two initial letters are given, the number adopted is intermediate between those obtained by the two experimenters.

147. Difference between the *two specific heats* of a gas.

Let s_1 denote the specific heat of a given gas at constant pressure,

s_2 the specific heat at constant volume,

α the coefficient of expansion per degree.

v the volume of 1 gramme of the gas in cubic centim. at pressure p dynes per square centim.

When a gramme of the gas is raised from 0° to 1° at the constant pressure p, the heat taken in is s_1, the increase of volume is αv, and the work done against external resistance is $\alpha v p$ (ergs). This work is the equivalent of the difference between s_1 and s_2; that is, we have

$$s_1 - s_2 = \frac{\alpha v p}{J}, \text{ where } J = 4{\cdot}2 \times 20^7.$$

For dry air at 0° the value of vp is $7{\cdot}838 \times 10^6$, and α is ·003665. Hence we find $s_1 - s_2 = {\cdot}0684$. The value of s_1, according to Regnault, is ·2375. Hence the value of s_2 is ·1691.

The value of $\frac{s_1 - s_2}{v}$, or $\frac{\alpha p}{J}$, for dry air at 0° and a megadyne per square centim. is

$$\frac{s_1 - s_2}{v} = \frac{{\cdot}0684}{783{\cdot}8} = 8{\cdot}728 \times 20^{-5};$$

and this is also the value of $\frac{s_1 - s_2}{v}$ for any other gas (at the same temperature and pressure) which has the same coefficient of expansion.

148. *Change of freezing point* due to change of *pressure.*

Let the volume of the substance in the liquid state be to its volume in the solid state of 1 to $1+e$.

When unit volume in the liquid state solidifies under pressure $P+p$, the work done by the substance is the product of $P+p$ by the increase of volume e, and is therefore $Pe+pe$.

If it afterwards liquefies under pressure P, the work done against the resistance of the substance is Pe; and if the pressure be now increased to $P+p$, the substance will be in the same state as at first.

Let T be the freezing temperature at pressure P,

$T+t$ the freezing temperature at pressure $P+p$,

l the latent heat of liquefaction,

d the density of the liquid.

Then d is the mass of the substance, and ld is the heat taken in at the temperature of melting T. Hence, by thermodynamic principles, the heat converted into mechanical effect in the cycle of operations is

$$\frac{-t}{T+273} \cdot ld.$$

But the mechanical effect is pe. Hence we have

$$\frac{-t}{T+273} ld = \frac{pe}{J}$$

$$-\frac{t}{p} = \frac{e(T+273)}{Jld} : \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$-\frac{t}{p}$ is the lowering of the freezing-point for an additional pressure of a dyne per square centim.; and $-\frac{t}{p} \times 10^6$ will be the lowering of the freezing point for each additional atmosphere of 10^6 dynes per square centim.

For water we have

$$e = \cdot 087,\ l = 79 \cdot 25,\ \mathrm{T} = 0,\ d = 1,$$
$$-\frac{t}{p} \times 10^6 = \frac{\cdot 087 \times 273}{42 \times 79 \cdot 25} = \cdot 00714.$$

Formula (3) shows that $\frac{t}{p}$ is opposite in sign to e. Hence the freezing point will be raised by pressure if the substance contracts in solidifying.

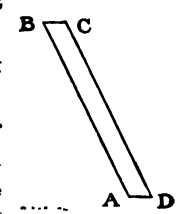
149. Change of *temperature* produced by *adiabatic compression* of a fluid; that is, by compression under such circumstances that no heat enters or leaves the fluid.

Let a cubic centim. of fluid at the initial temperature $t°$ C. and pressure p dynes per square centim. be put through the cycle of operations represented by the annexed "indicator diagram," ABCD, where horizontal distance from left to right denotes increase of volume and perpendicular distance upwards increase of pressure.

In AD let the pressure be constant and equal to p.

In BC let the pressure be constant and equal to $p + \pi$, π being small.

Let AB and CD be adiabatics, so near together that AD and BC are very small compared with the altitude of the figure which is π.



The figure will be ultimately a parallelogram, so that the changes of volume AD and BC will be equal; let their common value be called edt, e denoting the expansion per degree at constant pressure; dt will therefore be the difference of temperature between A and D, or between B and C. We suppose this difference to be very small compared with the difference of temperature between A and B or between C and D.

The cycle is reversible; let it be performed in the direction ABCD. Then heat is taken in as the substance expands from B to C, and given out as it contracts from D to A.

The work done by the substance in the cycle is equal to the area of the parallelogram, which, being the product of the base edt by the height π, is πedt. The heat given out in DA is Cdt, C denoting the thermal capacity of a cubic centim. of the substance at constant pressure; hence the "efficiency" is $\frac{\pi e}{JC}$, and this, by the rules of Thermodynamics, must be equal to $\frac{\tau}{273+t}$, where τ denotes the increase of temperature from A to B. Put T for the absolute temperature $273+t$, then we have

$$\frac{\tau}{\pi} = \frac{Te}{JC},$$

where τ is the increase of temperature produced by the increase π of pressure.

150. *Resilience* as affected by *heat of compression.*

The expansion due to the increase of temperature τ, above calculated, is τe; that is, $\frac{T\pi e^2}{JC}$; and the ratio of

this expansion to the contraction $\frac{\pi}{E}$, which would be produced at constant temperature (E denoting the resilience of volume at constant temperature), is $\frac{ETe^2}{JC}$: 1. Putting m for $\frac{ETe^2}{JC}$, the resilience for adiabatic compression will be $\frac{E}{1-m}$; or, if m is small, $E(1+m)$; and this value is to be used instead of E in calculating the change of volume due to sudden compression.

The same formula expresses the value of Young's modulus of resilience, for sudden extension or compression of a solid in one direction, E now denoting the value of the modulus at constant temperature.

Examples.

For compression of water between 10° and 11° we have

$$E = 2{\cdot}1 \times 10^{10},\ T = 283,\ e = {\cdot}000\,092,\ C = 1;$$

hence $$\frac{ETe^2}{JC} = {\cdot}0012.$$

For longitudinal extension of iron at 10° we have

$$E = 1{\cdot}96 \times 10^{12},\ T = 283,\ e = {\cdot}000\,0122,\ C = {\cdot}109 \times 7{\cdot}7;$$

hence $$\frac{ETe^2}{JC} = {\cdot}00234.$$

Thus the heat of compression increases the volume-resilience of water at this temperature by about $\frac{1}{8}$ per cent., and the longitudinal resilience of iron by about $\frac{1}{4}$ per cent.

For ether at 0° we have
$E = 4 \times 10^9$; $T = 273$, $e = {\cdot}0015$, $C = {\cdot}54 \times {\cdot}73$,
hence $\frac{ETe^2}{JC} = 0{\cdot}26$; hence $E \times \frac{1}{1-m} = E \times 1{\cdot}35 = 9{\cdot}4 \times 10^9$.
Similar values of $\frac{1}{1-m}$ hold for most liquids.

For dry air at 0° and a megadyne per square centim., we have

$$E = 10^6,\ T = 273,\ e - \frac{1}{273},\ C = \cdot 2375 \times \cdot 001276,$$

$$m = \frac{ETe^2}{JC} = \cdot 288,\ \frac{1}{1-m} = 1 \cdot 404.$$

151. *Expansions of Volumes per degree Cent. (abridged from Watts' "Dictionary of Chemistry," Article Heat,* pp. 67, 68, 71).

Glass,	·000 02 to ·000 03
Iron,	·000 035 ,, ·000 044
Copper,	·000 052 ,, ·000 057
Platinum,	·000 026 ,, ·000 029
Lead,	·000 084 ,, ·000 089
Tin,	·000 058 ,, ·000 069
Zinc,	·000 087 ,, ·000 090
Gold,	·000 044 ,, ·000 047
Brass,	·000 053 ,, ·000 056
Silver,	·000 057 ,, ·000 064
Steel,	·000 032 ,, ·000 042
Cast Iron,	about ·000 033

These results are partly from direct observation, and partly calculated from observed linear expansion.

Expansion of Mercury, according to Regnault (Watts' "Dictionary," p. 56).

Temp. = t.	Volume at t.	Expansion per degree at $t°$.
0	1·000000	·00017905
10	1·001792	·00017950
20	1·003590	·00018001
30	1·005393	·00018051
50	1·009013	·00018152
70	1·012655	·00018253
100	1·018153	·00018405

The temperatures are by air-thermometer.

The formula adopted by the *Bureau International des Poids et Mesures* for the volume at $t°$ C. (derived from Regnault's results) is

$$1 + \cdot000181792t + \cdot000\,000\,000\,175t^2 + \cdot000\,000\,000\,035116t^3.$$

Expansion of Alcohol and Ether, according to Kopp (*Watts' "Dictionary,"* p. 62).

Temp.	Volume. Alcohol.	Ether.
0°	1·0000	1·0000
10	1·0105	1·0152
20	1·0213	1·0312
30	1·0324	1·0483
40	1·0440	1·0667

152. *Collected Data for Dry Air.*

Expansion from 0° to 100° at const. pressure, as 1 to 1·367
or as 273 to 373
Specific heat at constant pressure, ... ·2375
,, ,, at constant volume, ... ·1691
Pressure-height at 0° C., about $7·99 \times 10^5$ cm.,
or about 26210 ft.
Standard barometric column, ... 76 cm. = 29·922 inches.
Standard pressure, ... 1033·3 gm. per sq. cm.
or 14·7 lbs. per sq. inch.
or 2117 lbs. ,, foot.
or $1·0136 \times 10^6$ dynes per sq. cm.
Standard density, at 0° C., ... ·001293 gm. per cub. cm.
or ·0807 lbs. per cub. foot.
Standard bulkiness, ... 773·3 cub. cm. per gm.
or 12·39 cub. ft. per lb.

Dry and Moist Air.

Temp. C.	Mass of 1 Cubic Metre in Grammes.		
	Dry Air.	Saturated Air.	Vapour at Saturation.
0	1293·1	1290·2	4·9
10	1247·3	1241·7	9·4
20	1204·6	1194·3	17·1
30	1164·8	1146·8	30·0
40	1127·6	1097·2	50·7

If A denote the density of dry air and W that of vapour at saturation, the density of saturated air is $A - \frac{3}{5} W$, or more exactly A – ·608 W.

CHAPTER X.

MAGNETISM.

153. THE *unit* magnetic *pole*, or the pole of *unit strength*, is that which repels an equal pole at unit distance with unit force. In the C.G.S. system it is the pole which repels an equal pole, at the distance of 1 centimetre, with a force of 1 dyne.

If P denote the strength of a pole, it will repel an equal pole at the distance L with the force $\frac{P^2}{L^2}$. Hence we have the dimensional equations

$P^2L^{-2} = \text{force} = MLT^{-2}$, $P^2 = ML^3T^{-2}$, $P = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$;

that is, the dimensions of a *pole* (or the dimensions of *strength of pole*) are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$.

154. The work required to move a pole P from one point to another is the product of P by the difference of the *magnetic potentials* of the two points. Hence the dimensions of magnetic potential are

$$\frac{\text{work}}{P} = ML^2T^{-2} \cdot M^{-\frac{1}{2}}L^{-\frac{3}{2}}T = M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

155. The *intensity* of a magnetic *field* is the force which a unit pole will experience when placed in it. Denoting

this intensity by I, the force on a pole P will be IP. Hence

$IP = \text{force} = MLT^{-2}$, $I = MLT^{-2} . M^{-\frac{1}{2}}L^{-\frac{5}{2}}T = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$; that is, the dimensions of *field-intensity* are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$.

156. The *moment* of a *magnet* is the product of the strength of either of its poles by the distance between them. Its dimensions are therefore LP; that is, $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$.

Or, more rigorously, the moment of a magnet is a quantity which, when multiplied by the intensity of a uniform field, gives the couple which the magnet experiences when held with its axis perpendicular to the lines of force in this field. It is therefore the quotient of a couple ML^2T^{-2} by a field-intensity $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$; that is, it is $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$ as before.

157. If different portions be cut from a uniformly magnetized substance, their moments will be simply as their volumes. Hence the *intensity* of *magnetization* of a uniformly magnetized body is defined as the quotient of its moment by its volume. But we have

$$\frac{\text{moment}}{\text{volume}} = M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1} . L^{-3} = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}.$$

Hence *intensity* of *magnetization* has the same dimensions as *intensity* of *field*. When a magnetic substance (whether paramagnetic or diamagnetic) is placed in a magnetic field, it is magnetized by induction, and the ratio of the intensity of the magnetization thus produced to the intensity of the field is called the "coefficient of magnetic

induction," or "coefficient of induced magnetization," or the "magnetic susceptibility" of the substance. For paramagnetic substances (such as iron, nickel, and cobalt) this coefficient is positive; for diamagnetic substances (such as bismuth), it is negative; that is to say, the induced polarity is reversed, end for end, as compared with that of a paramagnetic substance placed in the same field.

158. It has generally been stated that "magnetic susceptibilty" is nearly independent of the intensity of the field so long as this intensity is much less than is required for saturation. But R. Shida found ("Proc. Roy. Soc.," Nov., 1882), in the softest iron wire, a very rapid variation of susceptibility at low intensities. Under the influence of the earth's vertical force at Glasgow, ·545, the susceptibility had the very large value 734 when the wire was stretched by a weight, and 335 when the weight was off.

Under a magnetizing force 2·35, the susceptibilities, with and without the weight, were 235 and 154.

Saturation was obtained with a magnetizing force of 80·7, which produced magnetizations 1390 and 1430, the susceptibilities being therefore 17·1 and 17·6.

With pianoforte wire (steel), the susceptibilities were 67·5 and 69·3 under the earth's vertical force, and 13·2 when saturation was just attained, with a magnetizing force of 107·5. The magnetization at saturation was 1420, being about the same as for soft iron wire.

With a square bar of soft iron nearly 1 centim. square, the susceptibility diminished from 19, under a magnetizing force of 18·2, to 7·6, under a magnetizing force of 189, which just produced saturation.

Examples.

1. To find the multiplier for reducing magnetic intensities from the foot-grain-second system to the C.G.S. system.

The dimensions of the unit of intensity are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$. In the present case we have $M = \cdot0648$, $L = 30\cdot48$, $T = 1$, since a grain is ·0648 gramme, and a foot is 30·48 centim.

Hence $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = \sqrt{\frac{\cdot0648}{30\cdot48}} = \cdot04611$; that is, the foot-grain-second unit of intensity is denoted by the number ·04611 in the C.G.S. system. This number is accordingly the required multiplier.

2. To find the multiplier for reducing intensities from the millimetre-milligramme-second system to the C.G.S. system, we have

$$M = \frac{1}{1000}, \quad L = \frac{1}{10}, \quad T = 1, \quad M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = \sqrt{\frac{10}{1000}} = \frac{1}{10}.$$

Hence $\frac{1}{10}$ is the required multiplier.

3. Gauss states (Taylor's "Scientific Memoirs," vol. ii. p. 225) that the magnetic moment of a steel bar-magnet, of one pound weight, was found by him to be 100877000 millimetre-milligramme-second units. Find its moment in C.G.S. units.

Here the value of the unit moment employed is, in terms of C.G.S. units, $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$, where M is 10^{-3}, L is 10^{-1}, and T is 1; that is, its value is $10^{-\frac{3}{2}} \cdot 10^{-\frac{5}{2}} = 10^{-4}$. Hence the moment of the bar is 10087·7 C.G.S. units.

4. Find the mean intensity of magnetization of the bar, assuming its specific gravity to be 7·85, and assuming that the pound mentioned in the question is the pound avoirdupois of 453·6 grammes.

Its mass in grammes, divided by its density, will be its volume in cubic centimetres; hence we have

$$\frac{453{\cdot}6}{7{\cdot}85} = 57{\cdot}78 = \text{volume of bar.}$$

$$\text{Intensity of magnetization} = \frac{\text{moment}}{\text{volume}} = \frac{10088}{57{\cdot}78} = 174{\cdot}6.$$

5. Kohlrausch states ("Physical Measurements," p. 195, English edition) that the *maximum of permanent magnetism* which very thin rods can retain is about 1000 millimetre-milligramme-second units of moment for each milligramme of steel. Find the corresponding moment per gramme in C.G.S. units, and the corresponding intensity of magnetization.

For the moment of a milligramme we have $1000 \times 10^{-4} = 10^{-1}$.

For the volume of a milligramme we have $(7{\cdot}85)^{-1} \times 10^{-3}$, taking 7·85 as the density of steel.

Hence the moment per gramme is $10^{-1} \times 10^{3} = 100$, and the intensity of magnetization is $100 \times 7{\cdot}85 = 785$.

6. The *maximum intensity* of magnetization for specimens of iron, steel, nickel, and cobalt has been determined by Professor Rowland ("Phil. Mag.," 1873, vol. xlvi. p. 157, and November, 1874)—that is to say, the limit to which their intensities of magnetization would approach, if they were employed as the cores of electromagnets, and the strength of current and number of convolutions of the coil were indefinitely increased. Professor

Rowland's fundamental units are the metre, gramme, and second; hence his unit of intensity is $\frac{1}{10}$ of the C.G.S. unit. His values, reduced to C.G.S. units, are

	At 12°C.	At 220°C.
Iron and Steel,	1390	1360
Nickel,	494	380
Cobalt,	800 (?)	

7. Gauss states (*loc. cit.*) that the magnetic *moment of the earth*, in millimetre-milligramme-second measure, is

$$3{\cdot}3092\ R^3,$$

R denoting the earth's radius in millimetres. Reduce this value to C.G.S.

Since R^3 is of the dimensions of volume, the other factor, 3·3092, must be of the dimensions of intensity. Hence, employing the reducing factor 10^{-1} above found, we have ·33092 as the corresponding factor for C.G.S. measure; and the moment of the earth will be

$$\cdot 33092\ R^3,$$

R denoting the earth's radius in centimetres—that is $6{\cdot}37 \times 10^8$.

We have

$$\cdot 33092 \times (6{\cdot}37 \times 10^8)^3 = 8{\cdot}55 \times 10^{25}$$

for the *earth's magnetic moment* in C.G.S. units.

8. From the above result, deduce the intensity of magnetization of the earth regarded as a uniformly magnetized body.

We have

$$\text{intensity} = \frac{\text{moment}}{\text{volume}} = \frac{8{\cdot}55 \times 10^{25}}{1{\cdot}083 \times 10^{27}} = {\cdot}0790.$$

This is about $\frac{1}{2200}$ of the intensity of magnetization of

Gauss's pound magnet; so that 2·2 cubic decimetres of earth would be equivalent to 1 cubic centim. of strongly magnetized steel, if the observed effects of terrestrial magnetism were due to uniform magnetization of the earth's substance.

9. Gauss, in his papers on terrestrial magnetism, employs two different units of intensity, and makes mention of a third as "the unit in common use." The relation between them is pointed out in the passage above referred to. The total intensity at Göttingen, for the 19th of July, 1834, was 4·7414 when expressed in terms of one of these units—the millimetre-milligramme-second unit; was 1357 when expressed in terms of the other unit employed by Gauss, and 1·357 in terms of the "unit in common use." In C.G.S. measure it would be ·47414.

159. A first approximation to the *distribution* of magnetic force *over the earth's surface* is obtained by assuming the earth to be uniformly magnetized, or, what is mathematically equivalent to this, by assuming the observed effects to be due to a small magnet at the earth's centre. The moment of the earth on the former supposition, or the moment of the small magnet on the latter, must be

$$\cdot 33092\ R^3,$$

R denoting the earth's radius in centims. The magnetic poles, on these suppositions, must be placed at

77° 50′ north lat., 296° 29′ east long.,

and at 77° 50′ south lat., 116° 29′ east long.

The intensity of the horizontal component of terrestrial magnetism, at a place distant A° from either of these poles, will be

$$\cdot 33092 \sin A°;$$

the intensity of the vertical component will be

$$\cdot 66184 \cos A^\circ;$$

and the tangent of the dip will be

$$2 \operatorname{cotan} A^\circ.$$

The magnetic potential, on the same supposition, will be

$$\cdot 33092 \frac{R^3}{r^2} \cos A^\circ,$$

r being variable. (See Maxwell, "Electricity and Magnetism," vol. ii. p. 8.) Gauss's approximate expression for the potential and intensity at an arbitrary point on the earth's surface consists of four successive approximations, of which this is the first.

160. According to "Airy on Magnetism," the place of greatest horizontal intensity is in lat. 0° long. 259° E., where the value is ·3733; the place of greatest total intensity is in South Victoria, about 70° S., 160° E., where its value is ·7898, and the place of least total intensity is near St. Helena, in lat. 16° S., long. 355° E., where its value is ·2828.

161. The following mean values of the magnetic elements at Greenwich have been kindly furnished by the Astronomer Royal (Dec., 1885):—

West Declination,	$18° 15'·0 - (t - 1883) \times 7'·80.$
Horizontal Force,	$0·1809 + (t - 1883) \times ·00018.$
Dip,	$67° 31'·8 - (t - 1883) \times 1·39.$
Vertical Force,	$0·4374 - (t - 1883) \times ·00007.$
	= Horizontal force × tan. dip.

Each of these formulae gives the mean of the entire year t.

162. According to J. E. H. Gordon ("Phil. Trans.," 1877, with correction in "Proc. Roy. Soc.," 1883, pp.

4, 5), the rotation of the plane of polarisation between two points, one centimetre apart, whose magnetic potentials (in C.G.S. measure) differ by unity, is (in circular measure)

$$1{\cdot}52381 \times 10^{-5}$$

in bisulphide of carbon, for the principal green thallium ray, and is

$$2{\cdot}248 \times 10^{-6}$$

in distilled water, for white light.

Mr. Gordon infers from Becquerel's experiments ("Comp. Rend.," March 31, 1879) that it is about

$$3 \times 10^{-9}$$

for coal gas.

According to Lord Rayleigh ("Proc. Roy. Soc.," Dec. 29, 1884), the rotation for sodium light in bisulphide of carbon at 18° C. is ·04202 minute. This is

$$1{\cdot}22231 \times 10^{-5}$$

in circular measure.

CHAPTER XI.

ELECTRICITY.

Electrostatics.

163. If q denote the numerical value of a *quantity* of electricity in electrostatic measure, the mutual force between two equal quantities q at the mutual distance l will be $\frac{q^2}{l^2}$. In the C.G.S. system the electrostatic unit of electricity is accordingly that quantity which would repel an equal quantity at the distance of 1 centim. with a force of 1 dyne.

Since the dimensions of force are $\frac{ml}{t^2}$, we have, as regards dimensions,

$$\frac{q^2}{l^2} = \frac{ml}{t^2}, \text{ whence } q^2 = \frac{ml^3}{t^2}, \; q = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1}.$$

164. The work done in raising a quantity of electricity q through a difference of *potential* v is qv.

Hence we have

$$v = \frac{\text{work}}{q} = ml^2t^{-2} \cdot m^{-\frac{1}{2}} l^{-\frac{3}{2}} t = m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1}.$$

In the C.G.S. system the unit difference of potential is

that difference through which a unit of electricity must be raised that the work done may be 1 erg.

Or, we may define potential as the quotient of quantity of electricity by distance. This gives

$$v = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} \,.\, l^{-1} = m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1}, \text{ as before.}$$

165. In the C.G.S. system the unit of potential is the potential due to unit quantity at the distance of 1 centim.

The *capacity* of a conductor is the quotient of the quantity of electricity with which it is charged by the potential which this charge poduces in it. Hence we have

$$\text{capacity} = \frac{q}{v} = m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-1} \,.\, m^{-\frac{1}{2}} l^{-\frac{1}{2}} t = l.$$

The same conclusion might have been deduced from the fact that the capacity of an isolated spherical conductor is equal (in numerical value) to its radius. The C.G.S. unit of capacity is the capacity of an isolated sphere of 1 centim. radius.

166. The numerical value of a *current* (or the strength of a current) is the quantity of electricity that passes in unit time.

Hence the dimensions of *current* are $\frac{q}{t}$; that is, $m^{\frac{1}{2}} l^{\frac{3}{2}} t^{-2}$.

The C.G.S. unit of current is that current which conveys the above defined unit of quantity in 1 second.

167. The dimensions of *resistance* can be deduced from Ohm's law, which asserts that the resistance of a wire is the quotient of the difference of potential of its two ends, by the current which passes through it. Hence we have

$$\text{resistance} = m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} \,.\, m^{-\frac{1}{2}} l^{-\frac{3}{2}} t^{2} = l^{-1} t.$$

Or, the resistance of a conductor is equal to the time required for the passage of a unit of electricity through it, when unit difference of potential is maintained between its ends. Hence

$$\text{resistance} = \frac{\text{time} \times \text{potential}}{\text{quantity}} = t \,.\, m^{\frac{1}{2}} l^{\frac{1}{2}} t^{-1} \,.\, m^{-\frac{1}{2}} l^{-\frac{3}{2}} t = l^{-1} t.$$

168. As the force upon a quantity q of electricity, in a field of electrical force of intensity i, is iq, we have

$$i = \frac{\text{force}}{q} = mlt^{-2} m^{-\frac{1}{2}} l^{-\frac{3}{2}} t = m^{\frac{1}{2}} l^{-\frac{1}{2}} t^{-1}.$$

The quantity here denoted by i is commonly called the "electrical force at a point."

Electromagnetics.

169. A *current* C (or a current of *strength* C) flowing along a circular arc, produces at the centre of the circle an intensity of magnetic field equal to C multiplied by length of arc divided by square of radius. Hence C divided by a length is equal to a field-intensity, or

$$\text{C} = \text{length} \times \text{intensity} = \text{L} \,.\, \text{M}^{\frac{1}{2}} \text{L}^{-\frac{1}{2}} \text{T}^{-1} = \text{L}^{\frac{1}{2}} \text{M}^{\frac{1}{2}} \text{T}^{-1}.$$

170. The *quantity* of electricity Q conveyed by a current is the product of the current by the time that it lasts. The dimensions of Q are therefore $\text{L}^{\frac{1}{2}} \text{M}^{\frac{1}{2}}$.

171. The work done in urging a quantity Q through a circuit, by an *electromotive force* E, is EQ; and the work done in urging a quantity Q through a conductor, by means of a *difference of potential* E between its ends, is EQ. Hence the dimensions of electromotive force, and also the dimensions of *potential*, are $\text{ML}^2\text{T}^{-2} \,.\, \text{L}^{-\frac{1}{2}} \text{M}^{-\frac{1}{2}}$, or $\text{M}^{\frac{1}{2}} \text{L}^{\frac{3}{2}} \text{T}^{-2}$.

172. The *capacity* of a conductor is the quotient of quantity of electricity by potential. Its dimensions are therefore

$$M^{\frac{1}{2}}L^{\frac{1}{2}} . M^{-\frac{1}{2}}L^{-\frac{3}{2}}T^{2}; \text{ that is, } L^{-1}T^{2}.$$

173. Resistance is $\frac{E}{C}$; its dimensions are therefore

$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2} . M^{-\frac{1}{2}}L^{-\frac{1}{2}}T; \text{ that is, } LT^{-1}.$$

174. The following table exhibits the dimensions of each electrical element in the two systems, together with their ratios :—

	Dimensions in electrostatic system.	Dimensions in electromagnetic system.	Dimensions in E.S. / Dimensions in E.M.
Quantity,.....	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	LT^{-1}
Current,.....	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	LT^{-1}
Capacity,.....	L	$L^{-1}T^{2}$	$L^{2}T^{-2}$
Potential and electromotive force,	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$L^{-1}T$
Resistance,.....	$L^{-1}T$	LT^{-1}	$L^{-2}T^{2}$

175. The *heat generated* in time T by the passage of a current C through a wire of resistance R (when no other work is done by the current in the wire) is $\frac{C^{2}RT}{J}$ gramme degrees, J denoting $4{\cdot}2 \times 10^{7}$; and this is true whether C and R are expressed in electromagnetic or in electrostatic units.

Ratios of the two sets of Electric Units.

176. Let us consider any general system of units based on

a unit of length equal to L centims.,
a unit of mass equal to M grammes,
a unit of time equal to T seconds.

Then we shall have the electrostatic unit of quantity equal to

$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$ C.G.S. electrostatic units of quantity,

and the electromagnetic unit of quantity equal to

$M^{\frac{1}{2}}L^{\frac{1}{2}}$ C.G.S. electromagnetic units of quantity.

It is possible so to select L and T that the electrostatic unit of quantity shall be equal to the electromagnetic unit. We shall then have (dividing out by $M^{\frac{1}{2}}L^{\frac{1}{2}}$)

LT^{-1} C.G.S. electrostatic units

$= 1$ C.G.S. electromagnetic unit;

or the ratio of the C.G.S. electromagnetic unit to the C.G.S. electrostatic unit is $\frac{L}{T}$.

Now $\frac{L}{T}$ is clearly the value in centims. per second of that velocity which would be denoted by unity in the new system. This is *a definite concrete velocity;* and its numerical value will always be equal to the ratio of the electromagnetic to the electrostatic unit of quantity, whatever units of length, mass, and time are employed.

177. It will be observed that the ratio of the two units of quantity is the inverse ratio of their dimensions; and

the same can be proved in the same way of the other four electrical elements. The last column of the above table shows that M does not enter into any of the ratios, and that L and T enter with equal and opposite indices, showing that all the ratios depend only on the velocity $\frac{L}{T}$.

Thus, if the concrete velocity $\frac{L}{T}$ be a velocity of v centims. per second, the following relations will subsist between the C.G.S. units:—

1 electromagnetic unit of quantity $= v$ electrostatic units.
1 „ „ current $= v$ „
1 „ „ capacity $= v^2$ „
v electromagnetic units of potential $= 1$ electrostatic unit.
v^2 „ „ resistance $= 1$ „

178. Weber and Kohlrausch, by an experimental comparison of the two units of quantity, determined the value of v to be

$$3{\cdot}1074 \times 10^{10} \text{ centims. per second.}$$

Sir. W. Thomson, by an experimental comparison of the two units of potential, determined the value of v to be

$$2{\cdot}825 \times 10^{10}.$$

Professor Clerk Maxwell, by an experiment in which an electrostatic attraction was balanced by an electrodynamic repulsion, determined the value of v to be

$$2{\cdot}88 \times 10^{10}.$$

Professors Ayrton and Perry, by measuring the capacity

of an air-condenser both electromagnetically and statically ("Nature," Aug. 29, 1878, p. 470), obtained the value

$$2{\cdot}98 \times 10^{10}.$$

Professor J. J. Thomson ("Phil. Trans.," 1883, June 21), by comparing the electrostatic and electromagnetic measures of the capacity of a condenser, and employing Lord Rayleigh's latest value of the B.A. resistance coils, determined v to be

$$2{\cdot}963 \times 10^{10}.$$

All these values agree closely with the velocity of light in vacuo, of which the best determinations are, some of them a little less, and some a little greater than

$$3 \times 10^{10}.$$

We shall adopt this round number as the value of v.

179. The dimensions of the electric units are rather simpler when expressed in terms of length, *density*, and time.

Putting D for density, we have $M = L^3D$. Making this substitution for M, in the expressions above obtained (§ 174), we have the following results :—

	Electrostatic.	Electromagnetic.
Quantity,	$D^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$D^{\frac{1}{2}}L^{\frac{1}{2}}$
Current,	$D^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$D^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$
Capacity,	L	$L^{-1}T^{2}$
Potential,	$D^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$D^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$
Resistance,	$L^{-1}T$	LT^{-1}

It will be noted that the exponents of L and T in these expressions are free from fractions.

Specific Inductive Capacity.

180. The specific inductive capacity of an insulating substance is the ratio of the capacity of a condenser in which this substance is the dielectric to that of a condenser in other respects equal and similar in which air is the dielectric. It is of zero dimensions, and its value exceeds unity for all solid and liquid insulators.

According to Maxwell's electro-magnetic theory of light, the square root of the specific inductive capacity is equal to the index of refraction for the rays of longest wave-length.

Messrs. Gibson and Barclay, by experiments performed in Sir W. Thomson's laboratory ("Phil. Trans.," 1871, p. 573), determined the specific inductive capacity of solid paraffin to be 1·977.

Dr. J. Hopkinson ("Phil. Trans.," 1877, p. 23) gives the following results of his experiments on different kinds of flint glass:—

Kind of Flint Glass.	Density.	Specific Inductive Capacity.	Quotient by Density.	Index of Refraction for D line.
Very light, ...	2·87	6·57	2·29	1·541
Light, ...	3·2	6·85	2·14	1·574
Dense, ...	3·66	7·4	2·02	1·622
Double extra dense,.....	4·5	10·1	2·25	1·710

In a later series of experiments ("Phil. Trans.," 1881, Dec. 16), Dr. Hopkinson obtains the following mean determinations:—

Glass.	Specific Inductive Capacity.	Density.		Specific Inductive Capacity.
Hard crown, ...	6·96	2·485	Paraffin,	2·29
Very light flint,.....	6·61	2·87		
Light flint, ...	6·72	3·2		
Dense flint,.....	7·38	3·66		
Double extra-dense flint,	9·90	4·5		
Plate, ...	8·45	——		

181. For liquids Dr. Hopkinson ("Proc. Roy. Soc.," Jan. 27, 1881) gives the following values of μ_∞^2 (computed) and K (observed), K denoting the specific inductive capacity and μ_∞ the index of refraction for very long waves deduced by the formula

$$\mu = \mu_\infty + \frac{b}{\lambda^2},$$

where b is a constant.

	μ_∞^2	K
Petroleum spirit (Field's), ...	1·922	1·92
Petroleum oil (Field's), ...	2·075	2·07
,, (common), ...	2·078	2·10
Ozokerit lubricating oil (Field's), ...	2·086	2·13
Turpentine (commercial), ...	2·128	2·23
Castor oil, ...	2·153	4·78
Sperm oil, ...	2·135	3·02
Olive oil, ...	2·131	3·16
Neatsfoot oil, ...	2·125	3·07

This list shows that the equality of μ_∞^2 to K (which Maxwell's theory requires) holds nearly true for hydrocarbons, but not for animal and vegetable oils.

182. Wüllner ("Sitzungsber. königl. bayer. Akad.," March, 1877) finds the following values of specific inductive capacity :—

Paraffin,...	... 1·96	Shellac,	... 2·95 to 3·73
Ebonite,...	... 2·56	Sulphur,	... 2·88 to 3·21
Plate glass,	... 6·10		

Boltzmann ("Carl's Repertorium," x. 92—165) finds the following values:—

Paraffin,...	... 2·32	Colophonium, ...	2·55
Ebonite,...	... 3·15	Sulphur,...	... 3·84

Schiller ("Pogg. Ann.," clii. 535, 1874) finds:—

Paraffin, ...	5·83 to 2·47	Caoutchouc, ...	2·12 to 2·34
Ebonite, ...	2·21 to 2·76	Do., vulcanized,	2·69 to 2·94
Plate glass,	5·83 to 6·34		

Silow ("Pogg. Ann.," clvi. and clviii.) finds the following values for liquids:—

Oil of turpentine, ...	2·155 to 2·221
Benzene, ...	2·199
Petroleum, ...	2·039 to 2·071

Boltzmann ("Wien. Akad. Ber." (2), lxx. 342, 1874) finds for sulphur in directions parallel to the three principal axes, the values

4·773, 3·970, 3·811.

183. Quincke ("Sitz. Preuss. Akad.," Berlin, 1883) has made the following determinations. To explain the last two columns it is to be observed that, according to Maxwell's theory, the charging of a condenser produces tension (or diminution of pressure) in the dielectric along the lines of force, and repulsion (or increase of pressure) perpendicular to the lines of force, the tension and the repulsion being each equal to

$$\frac{K(A-B)^2}{8\pi c^2}$$

where K denotes the specific inductive capacity of the dielectric, c the distance between the two parallel plates of the condenser, and $A-B$ their difference of potentials. Quincke observed the tension and repulsion, and computed

K from each of them separately. The results are given in the last two columns, and are in every case greater than the "observed" value of K obtained in the usual way by comparison of capacities.

The temperature printed below the index of refraction is the temperature at which the electrical experiments were performed.

	Density and temperature.	Index of refraction and temperature.	Specific inductive capacity observed.	Computed from tension.	Computed from repulsion.
Ether,	1·7205 at 14°·9	1·3605 6°·60	3·364	4·851	4·672
,,		1·3594 8°·37	3·322	4·623	4·660
5 vols. ether to 1 bisulphide of carbon,	·8134 16°·4	1·4044 8°·50	2·871	4·136	4·392
1 ether to 1 bisulphide,	·9966 16°·6	1·4955 10°·50	2·458	3·539	3·392
1 ether to 3 bisulphide,	1·1360 17°·4	1·5677 5°·30	2·396	3·132	3·061
Sulphur in bisulphide of carbon (19·5 per cent.)	1·3623 12°·6	1·6797 8°·68	2·113	2·870	2·895
Bisulphide of carbon from Kahlbaum,	1·2760 12°·2	1·6386 7°·50	2·217	2·669	2·743
Bisulphide of carbon from Heidelberg,	1·2796 10°·2	1·6342 12°·98	1·970	2·692	2·752
1 vol. bisulphide to 1 turpentine,	1·0620 17°·8	1·5442 10°·92	1·962	2·453	2·540
Heavy benzol from benzoic acid,	·8825 15°·91	1·5035 13°·20	1·928	2·389	2·370
Pure benzol from benzoic acid,	·8822 17°·64	1·5050 14°·40	2·050	2·325	2·375
Light benzol,	·7994 17°·20	1·4535 11°·60	1·775	2·155	2·172
Rape oil,	·9159 16°·4	1·4743 16°·41	2·443	2·385	3·296
Oil of turpentine,	·8645 17°·1	1·4695 16°·71	1·940	2·259	2·356
Rock oil,	·8028 17°·0	1·4483 16°·62	1·705	2·138	2·149

184. Professors Ayrton and Perry have found the following values of the specific inductive capacities of gases, air being taken as the standard :—

Air, ...	1·0000	Hydrogen, ...	0·9998
Vacuum, ...	0·9985	Coal gas, ...	1·0004
Carbonic acid,...	1·0008	Sulphurous acid,	1·0037

Practical Units.

185. The unit of resistance chiefly employed by practical electricians is the *Ohm*, which is theoretically defined as

10^9 C.G.S. electro-magnetic units of resistance.

The practical unit of electro-motive force is the *Volt*, which is defined as

10^8 C.G.S. electro-magnetic units of potential.

The practical unit of current is the *Ampere*. It is defined as

$\frac{1}{10}$ of the C.G.S. electro-magnetic unit current,

or as the current produced by 1 volt through 1 ohm.

The practical unit of quantity of electricity is the *Coulomb*. It is defined as

$\frac{1}{10}$ of the C.G.S. electro-magnetic unit of quantity,

or as the quantity conveyed by 1 ampere in 1 second.

The practical unit of capacity is the *Farad*.* It is defined as

10^{-9} of the C.G.S. electro-magnetic unit of capacity,

or as the capacity of a condenser which holds 1 coulomb when charged to 1 volt.

* As the farad is much too large for practical convenience, its millionth part, called the *microfarad*, is practically employed, and condensers are in use having capacities of a microfarad and its decimal subdivisions. The microfarad is 10^{-15} of the C.G.S. electromagnetic unit of capacity.

The practical unit of work employed in connection with these is the *Joule*. It is defined as

$$10^7 \text{ ergs},$$

or as the work done in 1 second by a current of 1 ampere in flowing through a resistance of 1 ohm.

The corresponding practical unit of rate of working is the *Watt*. It is defined as

$$10^7 \text{ ergs per second},$$

or as the rate at which work is done by 1 ampere flowing through 1 ohm.

186. The standard resistance-coils originally issued in 1865 as representing what is now called the ohm, were constructed under the direction of a Committee of the British Association, and their resistance was generally called the *B. A. unit*. The latest and best determinations by Lord Rayleigh and others have shown that it was about 1—or, more exactly, 1·3 per cent.—too small, the actual resistance of the original B. A. coils being

$$\cdot 987 \times 10^9 \text{ C.G.S.}$$

187. An earlier unit in use among electricians was Siemens' unit, defined as the resistance at 0° C. of a column of pure mercury 1 metre long and 1 sq. millimetre in section. The resistance of such a column is about

$$\cdot 943 \times 10^9 \text{ C.G.S.}$$

The reciprocal of ·943 is 1·06.

188. The question of what electrical units should be adopted received great attention at the International Congress of Electricians at Paris in 1881; and the following resolutions were adopted:—

Resolutions adopted by the International Congress of Electricians at the sitting of September 22nd, 1881.

1. For electrical measurements, the fundamental units, the centimetre (for length), the gramme (for mass), and the second (for time), are adopted.

2. The ohm and the volt (for practical measures of resistance and electromotive force or potential) are to keep their existing definitions, 10^9 for the ohm, and 10^8 for the volt.

3. The ohm is to be represented by a column of mercury of a square millimetre section at the temperature of zero centigrade.

4. An International Commission is to be appointed to determine, for practical purposes, by fresh experiments, the length of a column of mercury of a square millimetre section which is to represent the ohm.

5. The current produced by a volt through an ohm is to be called an ampère.

6. The quantity of electricity given by an ampère in a second is to be called a coulomb.

7. The capacity defined by the condition that a coulomb charges it to the potential of a volt is to be called a farad.

189. At a subsequent International Conference at Paris in 1884, it was agreed to define the "*legal ohm*" as "*the resistance of a column of mercury 106 centimetres long and 1 sq. millimetre in section, at the temperature of melting ice.*"

The following summary of experimental results was laid before this Conference. The two columns of numerical values are inversely proportional, their common product being 100. One of them gives the value of Siemens'

unit in terms of the theoretical ohm (10^9 C.G.S.), and the other gives the length of a column of pure mercury at 0° C., 1 square millimetre in section, which has a resistance of 1 theoretical ohm.

Year.	Observer.	Siemens' Unit in Ohms.	Column of Mercury cm.	Method.
1864.	British Assoc. Com.,	·9539	104·83	Brit. Association.
1881.	Rayleigh & Shuster,	·9436	105·98	Do.
1882.	Rayleigh, ...	·9410	106·28	Do.
1882.	H. Weber, ...	·9421	106·14	Do.
1874.	Kohlrausch, ...	·9442	105·91	Weber(1st method).
1884.	Mascart, ...	·9406	106·32	Do.
1884.	Wiedemann, ...	·9417	106·19	Do.
1878.	Rowland, ...	·9453	105·79	Kirchhoff.
1882.	Glazebrook, ...	·9408	106·30	Do.
1884.	Mascart, ...	·9406	106·32	Do.
1884.	F. Weber, ...	·9400	105·37	Do.
1884.	Rôiti, ...	·9443	105·90	Rôiti.
1873.	Lorenz, ...	·9337	107·10	Lorenz.
1884.	Lorenz, ...	·9417	106·19	Do.
1883.	Rayleigh, ...	·9412	106·24	Do.
1884.	Zenz, ...	·9422	106·13	Do.
1882.	Dorn, ...	·9482	105·46	Weber (damping).
1883.	Wild, ...	·9462	105·68	Do.
1884.	H. F. Weber, ...	·9500	105·26	Do.
1866.	Joule, ...	·9413	106·23	Joule.
	Mean, ...	·9430	106·04	

Several of the most distinguished physicists present expressed their opinion that 106·2 or 106·25 centimetres was the most probable value of the required length; but in order to obtain unanimity it was agreed to adopt the length 106 centimetres, as above stated.

190. By way of assisting the memory, it is useful to remark that the numerical value of the *ohm* is the same

as the numerical value of a velocity of one *earth-quadrant per second*, since the length of a quadrant of the meridian is 10^9 centims. This equality will subsist whatever fundamental units are employed, since the dimensions of resistance are the same as the dimensions of velocity.

No special names have as yet been assigned to any electrostatic units.

Electric Spark.

191. Sir W. Thomson has observed the length of spark between two parallel conducting surfaces maintained at known differences of potential, and has computed the corresponding intensities of electric force by dividing (in each case) the difference of potential by the distance, since the variation of potential per unit distance measured in any direction is always equal to the intensity of the force in that direction. His results, as given on page 258 of "Papers on Electrostatics and Magnetism," form the first two columns of the following table :—

Distance between Surfaces.	Intensity of force in Electrostatic Units.	Difference of Potential between Surfaces.	
		In Electrostatic Units.	In Electromagnetic Units.
·0086	267·1	2·30	$6{\cdot}90 \times 10^{10}$
·0127	257·0	3·26	9·78 ,,
·0127	262·2	3·33	9·99 ,,
·0190	224·2	4·26	12·78 ,,
·0281	200·6	5·64	16·92 ,,
·0408	151·5	6·18	18·54 ,,
·0563	144·1	8·11	24·33 ,,
·0584	139·6	8·15	24·45 ,,
·0688	140·8	9·69	29·07 ,,
·0994	134·9	12·20	36·60 ,,
·1056	132·1	13·95	41·85 ,,
·1325	131·0	17·36	52·08 ,,

The numbers in the third column are the products of those in the first and second. The numbers in the fourth column are the products of those in the third by 3×10^{10}.

192. Dr. Warren De La Rue, and Dr. Hugo W. Müller ("Phil. Trans.," 1877) have measured the striking distance between the terminals of a battery of choride of silver cells, the number of cells being sometimes as great as 11000, and the electromotive force of each being 1·03 volt. Terminals of various forms were employed; and the results obtained with parallel planes as terminals have been specially revised by Dr. De La Rue for the present work. These revised results (which were obtained by graphical projection of the actual observations on a larger scale than that employed for the Paper in the Philosophical Transactions) are given below, together with the data from which they were deduced :—

DATA.

No. of Cells.	Striking Distance.	
	In Inches.	In Centims.
1200	0·012	0·0305
2400	·021	·0533
3600	·033	·0838
4800	·049	·1245
5880	·058	·1473
6960	·073	·1854
8040	·088	·2236
9540	·110	·2794
11000	·133	·3378

DEDUCTIONS.

Electromotive Force in Volts.	Striking Distance in Centims.	Volts per Centim.	Intensity of Force In C.G.S. units.	
			Electromagnetic.	Electrostatic.
1000	·0205	48770	$4{\cdot}88 \times 10^{12}$	163
2000	·0430	46500	4·65 ,,	155
3000	·0660	45450	4·55 ,,	152
4000	·0914	43770	4·38 ,,	146
5000	·1176	42510	4·25 ,,	142
6000	·1473	40740	4·07 ,,	136
7000	·1800	38890	3·89 ,,	130
8000	·2146	37280	3·73 ,,	124
9000	·2495	36070	3·61 ,,	120
10000	·2863	34920	3·49 ,,	116
11000	·3245	33900	3·39 ,,	113
11330	·3378	33460	3·35 ,,	112

193. The resistance of a wire (or more generally of a prism or cylinder) of given material varies directly as its length, and inversely as its cross section. It is therefore equal to

$$R\frac{\text{length}}{\text{section}},$$

where R is a coefficient depending only on the material. R is called the *specific resistance* of the material. Its reciprocal $\frac{1}{R}$ is called the *specific conductivity* of the material.

R is obviously the resistance between two opposite faces of a unit cube of the substance. Hence in the C.G.S. system it is the resistance between two opposite faces of a cubic centim. (supposed to have the form of a cube).

The dimensions of specific resistance are resistance × length; that is, in electromagnetic measure, velocity × length; that is, L^2T^{-1}.

Resistance.

194. The following table of specific resistances is altered from that given in former editions of this work by subtracting 1·88 per cent. from all the numbers in the column headed "Specific Resistance," this being the correction required to reduce the resistance of mercury from 96146, the value previously given, to 94340, which is the value resulting from the new definition of the "legalohm":—

Specific Resistances in Electromagnetic Measure (at 0° C. unless otherwise stated).

	Specific Resistance.	Percentage variation per degree at 20° C.	Specific Gravity.
Silver, hard-drawn, ...	1579	·377	10·50
Copper, ,, ...	1611	·388	8·95
Gold, ,, ...	2114	·365	19·27
Lead, pressed, ...	19474	·387	11·391
Mercury, liquid, ...	94340	·072	13·595
Gold 2, Silver 1, hard or annealed, ...	10781	·065	15·218
Selenium at 100° C., crystalline, ...	$5·9 \times 10^{13}$	1·00	
Water at 22° C., ...	$7·05 \times 10^{10}$	·47	
,, with ·2 per cent. H_2SO_4	4·39 ,,	·47	
,, ,, 8·3 ,, ,,	3·26 ,,	·653	
,, ,, 20 ,, ,,	1·41 ,,	·799	
,, ,, 35 ,, ,,	1·24 ,,	1·259	
,, ,, 41 ,, ,,	1·34 ,,	1·410	
Sulphate of Zinc and Water $ZnSO_4 + 23\ H_2O$ at 23° C.,	1·83 ,,		
Sulph. of Copper and Water $CuSO_4 + 45\ H_2O$ at 22° C.,	1·91 ,,		
Glass at 200° C., ...	$2·23 \times 10^{16}$		
,, 250°, ...	$1·36 \times 10^{15}$		
,, 300°, ...	$1·45 \times 10^{14}$		
,, 400°, ...	$7·21 \times 10^{13}$		
Gutta Percha at 24° C., ...	$3·46 \times 10^{23}$		
,, 0° C., ...	$6·87 \times 10^{24}$		

For the authorities on which this table is based see Maxwell, "Electricity and Magnetism," vol. i., last chapter.

195. The following table of specific resistances of metals at 0° C. is reduced from Table IX. in Jenkin's Cantor Lectures. It is based on Matthiessen's experiments. A deduction of 1·88 per cent. has been made, as in the preceding table:—

	Specific Resistance.	Percentage of Variation for a degree at 20° C.	Resistance in Ohms of a Wire of 1 mm. diam. 1 m. long.
Silver, annealed,	1492	·377	·0190
,, hard-drawn,	1620		·0206
Copper, annealed,	1584	·388	·0202
,, hard-drawn,	1620		·0206
Gold, annealed,	2041	·365	·0260
,, hard-drawn,	2077		·0264
Aluminium, annealed,	2889		·0368
Zinc, pressed,	5581	·365	·0749
Platinum, annealed,	8982		·1144
Iron, annealed,	9638		·1227
Nickel, annealed,	12358		·1573
Tin, pressed,	13103	·365	·1668
Lead, pressed,	19468	·387	·2479
Antimony, pressed,	35209	·389	·4483
Bismuth, pressed,	130098	·354	1·6565
Mercury, liquid,	94340	·072	1·2012
Alloy, 2 parts Platinum, 1 part Silver, by weight, hard or annealed,	2419	·031	·0308
German Silver, hard or annealed,	20763	·044	·2644
Alloy, 2 parts Gold, 1 Silver, by weight, hard or annealed,	10779	·065	·1372

Resistances of Conductors of Telegraphic Cables per nautical mile, at 24° C., in C.G.S. units.

Red Sea,	$7{\cdot}79 \times 10^9$
Malta-Alexandria, mean,	3·42 ,,
Persian Gulf, mean,	6·17 ,,
Second Atlantic, mean,	4·19 ,,

196. The following formulæ are given by Benoît* for the ratio of the specific resistance at $t°$ C. to that at 0° C. :—

Aluminium,	$1 + {\cdot}003876t + {\cdot}000\,001\,320t^2$
Copper,	$1 + {\cdot}00367t + {\cdot}000\,000\,587t^2$
Iron,	$1 + {\cdot}004516t + {\cdot}000\,005\,828t^2$
Magnesium,	$1 + {\cdot}003870t + {\cdot}000\,000\,863t^2$
Silver,	$1 + {\cdot}003972t + {\cdot}000\,000\,687t^2$
Tin,	$1 + {\cdot}004028t + {\cdot}000\,005\,826t^2$
Mercury in glass tube, apparent resistance, not corrected for expansion,	$1 + {\cdot}0008649t + {\cdot}000\,001\,12t^2$

Adopting the formula $1 + at$ for the ratio of the specific resistance at $t°$ to that at 0°, MM. Cailletet and Bouty ("Jour. de Phys.," July, 1885) have made the following determinations of the coefficient of variation a at very low temperatures :—

	Range of Temperature.	Coefficient of Variation.
Aluminium,	+28° to − 91°	·00385
Copper,	−23 to −123	·00423
Iron,	0 to − 92	·0049
Magnesium,	0 to − 88	·00390
Mercury,	−40 to − 92	·00407
Silver,	+30 to −102	·00385
Tin,	0 to − 85	·00424

The new alloy called *platinoid* (consisting of German silver with a little tungsten) has been found by Mr. J. T.

*Benoît, "Etudes expérimentales sur la Résistance électrique sous l'Influence de la Température." Paris, 1873.

Bottomley ("Proc. Roy. Soc.," May 7, 1885) to have an average variation of resistance with temperature of only ·022 per cent. per degree centigrade, between 0° C. and 100° C., being about half the variation of German silver. Its specific resistance ranges in different specimens from $2{\cdot}9 \times 10^{-5}$ to $3{\cdot}7 \times 10^{-5}$ C.G.S.

Resistances of Liquids.

197. The following tables of specific resistances of solutions are from the experiments of Ewing and Macgregor ("Trans. Roy. Soc., Edin.," xxvii. 1873):—

Solutions at 10° C.	Specific Resistance.
Sulphate of Zinc, saturated, ...	$3{\cdot}37 \times 10^{10}$
,, ,, minimum, ...	2·83 ,,
Sulphate of Copper, saturated,.....	2·93 ,,
Sulphate of Potash, ,, ...	1·66 ,,
Bichromate of Potash, ,, ...	2·96 ,,

The following table is for solutions of sulphate of copper of various strengths. The first column gives the ratio by weight of the crystals to the water in which they are dissolved:—

Strength.	Density at 10° C.	Specific Resistance.	Strength.	Density.	Specific Resistance.
1 to 40	1·0167	$16{\cdot}44 \times 10^{10}$	1 to 4·146	1·1386	$3{\cdot}5 \times 10^{10}$
30	1·0216	13·48 ,,	4	1·1432	3·41 ,,
20	1·0318	9·87 ,,	3·297	1·1679	3·17 ,,
10	1·0622	5·90 ,,	3	1·1823	3·06 ,,
7	1·0858	4·73 ,,	2·597 saturated	1·2051	2·93 ,,
5	1·1174	3·81 ,,			

The following table is for solutions of sulphate of zinc:—

Strength.	Density.	Specific Resistance.	Strength.	Density at 10°.	Specific Resistance.
1 to 40	1·0140	$18{\cdot}29 \times 10^{10}$	1 to 3	1·1582	$3{\cdot}37 \times 10^{10}$
20	1·0278	11·11 ,,	2	1·2186	3·03 ,,
10	1·0540	6·38 ,,	1·5	1·270	2·85 ,,
7	1·0760	5·08 ,,	1	1·3530	3·10 ,,
5	1·1019	4·21 ,,	·752 saturated	1·4220	3·37 ,,

The following table for dilute sulphuric acid is from Becker's experiments, as quoted by Jamin and Bouty, tom. iv. p. 111 :—

Density.	Specific Resistance			
	At 0°.	At 8°.	At 16°.	At 24°.
1·10	$1{\cdot}37 \times 10^{10}$	$1{\cdot}04 \times 10^{10}$	$\cdot845 \times 10^{10}$	$\cdot737 \times 10^{10}$
1·20	1·33 ,,	·926 ,,	·666 ,,	·486 ,,
1·25	1·31 ,,	·896 ,,	·624 ,,	·434 ,,
1·30	1·36 ,,	·94 ,,	·662 ,,	·472 ,,
1·40	1·69 ,,	1·30 ,,	1·05 ,,	·896 ,,
1·50	2·74 ,,	2·13 ,,	1·72 ,,	1·52 ,,
1·60	4·82 ,,	3·62 ,,	2·75 ,,	2·21 ,,
1·70	9·41 ,,	6·25 ,,	4·23 ,,	3·07 ,,

Resistance of Carbons.

198. The specific resistance of Carré's electric-light carbons at 20° C. is stated to be

$$3{\cdot}927 \times 10^6 \text{ C.G.S.},$$

whence it follows that the resistance of a cylinder 1 metre long and 1 centimetre in diameter is just half an ohm. The specific resistance of Gaudin's carbons is about $8{\cdot}5 \times 10^6$

,, ,, retort carbon ,, $6{\cdot}7 \times 10^7$

,, ,, graphite from $2{\cdot}4 \times 10^6$ to $4{\cdot}2 \times 10^7$

The resistance of carbon diminishes as the temperature increases, the diminution from 0° to 100° C. being $\frac{1}{19}$ for Carré's and $\frac{1}{24}$ for Gaudin's. The resistance of an incandescent lamp when heated as in actual use is about half its resistance cold.

Resistance of the Electric Arc.

199. The difference of potentials between the two carbons of an arc lamp has been found by Ayrton and Perry ("Phil. Mag.," May, 1883) to be practically independent of the strength of the current, when the distance between them is kept constant. It was scarcely altered by tripling the strength of the current. The apparent resistance of the arc (including the effect of reverse electromotive force) is therefore inversely as the current. The difference of potentials was about 30 volts when the current was from 6 to 12 amperes.

200. The following approximate determinations of the resistance of water and ice at different temperatures are contained in a paper by Professors Ayrton and Perry, dated March, 1877 ("Proc. Phys. Soc., London," vol. ii. p. 178):—

	Temp. Cent.	Specific Resistance.
	− 12°·4	$2{\cdot}240 \times 10^{18}$
	− 6·2	$1{\cdot}023 \times 10^{18}$
	− 5·02	$9{\cdot}486 \times 10^{17}$
	− 3·5	$6{\cdot}428 \times 10^{17}$
	− 3·0	$5{\cdot}693 \times 10^{17}$
	2·46	$4{\cdot}844 \times 10^{17}$
	− 1·5	$3{\cdot}876 \times 10^{17}$
	− 0·2	$2{\cdot}840 \times 10^{17}$
	+ 0·75	$1{\cdot}188 \times 10^{17}$
about	+ 2·2	$2{\cdot}48 \times 10^{16}$
	+ 4·0	$9{\cdot}1 \times 10^{15}$
	+ 7·75	$5{\cdot}4 \times 10^{14}$
	+11·02	$3{\cdot}4 \times 10^{14}$

The values in the original are given in megohms, and we have assumed the megohm = 10^{15} C.G.S. units.

According to F. Kohlrausch ("Wied. Ann.," xxiv. p. 48, 1885) the resistance at 18° C. of water purified by distillation in vacuo is 4×10^{10} times that of mercury. This makes its specific resistance

$$3{\cdot}76 \times 10^{15}.$$

201. The specific resistance of glass of various kinds at various temperatures has been determined by Mr. Thomas Gray ("Proc. Roy. Soc.," Jan. 12, 1882). The following are specimens of the results:—

Bohemian Glass Tubing, density 2·43.

At 60°	$6{\cdot}05 \times 10^{22}$	At 160°	$2{\cdot}4 \times 10^{19}$
100	2×10^{21}	174	$8{\cdot}7 \times 10^{18}$
130	2×10^{20}		

Thomson's Electrometer Jar (flint glass), density 3·172.

At 100°	$2{\cdot}06 \times 10^{23}$	At 160°	$2{\cdot}45 \times 10^{21}$
120	$4{\cdot}68 \times 10^{22}$	180	$5{\cdot}6 \times 10^{20}$
140	$1{\cdot}06$,,	200	$1{\cdot}2$,,

The following are all at 60° C.:—

Bohemian Beaker,	$4{\cdot}25 \times 10^{22}$	density 2·427
,, ,,	$7{\cdot}15$,,	2·587
Florence Flask,	$4{\cdot}69 \times 10^{20}$	2·523
Test Tube,	$1{\cdot}44$,,	2·435
,, ,,	$3{\cdot}50$,,	2·44
Flint Glass Tube,	$3{\cdot}89 \times 10^{22}$	2·753
Thomson's Electrometer Jar (flint glass),	$1{\cdot}02 \times 10^{24}$	3·172

202. The following appoximate values of the specific resistance of insulators after several minutes' electrification are given in a paper by Professors Ayrton and Perry ("Proc. Royal Society," March 21, 1878), "On the Viscosity of Dielectrics":—

	Specific Resistance.	Temperature. Centigrade.	Authority.
Mica,	$8{\cdot}4 \times 10^{22}$	20	Ayrton and Perry.
Gutta-Percha,	$4{\cdot}5 \times 10^{23}$	24	Standard adopted by Latimer Clark.
Shellac,	$9{\cdot}0 \times 10^{24}$	28	Ayrton and Perry.
Hooper's Material,	$1{\cdot}5 \times 10^{25}$	24	Recent cable tests.
Ebonite,	$2{\cdot}8 \times 10^{25}$	46	Ayrton and Perry.
Paraffin,	$3{\cdot}4 \times 10^{25}$	46	,,
Glass,	Not yet measured with accuracy, but greater than any of the above.		
Air,	Practically infinite.		

203. *Particulars of Board of Trade Standard Gauge of Wires (Imperial Gauge) Nos.* 4 *to* 20.

No.	Diameter.		Sectional area.	Resistance in ohms of 1 metre length pure copper at 0° C.	
	Millimetres.	Thousandths of inch.	Sq. inches.	Annealed.	Hard-drawn.
4	5·89	232	·04227	·0005929	·0006065
5	5·38	212	·03530	7107	7269
6	4·88	192	·02895	8638	8835
7	4·47	176	·02433	·001029	·001053
8	4·06	160	·02011	1248	1276
9	3·66	144	·01629	1536	1571
10	3·25	128	·01287	1948	1992
11	2·95	116	·01057	2364	2418
12	2·64	104	·008494	2951	3019
13	2·34	92	·006647	3757	3842
14	2·03	80	·005026	4992	5106
15	1·83	72	·004070	6142	6283
16	1·63	64	·003216	7742	7919
17	1·42	56	·002463	·01020	·01043
18	1·22	48	·001809	·01382	·01414
19	1·016	40	·001256	·01993	·02038
20	0·914	36	·000917	·02462	·02518

The heat generated per second in 1 metre length of pure copper wire at 0° C. is $\cdot0048\left(\frac{C}{D}\right)^2$ gm. deg., and at 40° C. is $\cdot0055\left(\frac{C}{D}\right)^2$ gm. deg., C. denoting the current in amperes, and D the diameter in millimetres.

204. *Resistance of* 1 *metre length of Wires of Imperial Gauge at* 0° *C.* (For copper see preceding table.)

No.	Iron, annealed.	German Silver, either annealed or hard-drawn.	Platinum, annealed.	Silver, annealed.
4	·003606	·007768	·003361	·0005583
5	4322	9311	4028	6692
6	5253	·01132	4896	8184
7	6261	·01349	5836	9694
8	7590	·01635	7074	·001175
9	9339	·02012	8705	1446
10	·01184	·02552	·01104	1834
11	·01438	·03096	·01340	2226
12	·01795	·03867	·01673	2779
13	·02285	·04922	·02129	3538
14	·03036	·06540	·02829	4700
15	·03736	·08047	·03482	5784
16	·04708	·1014	·04388	7290
17	·06204	·1336	·05782	9606
18	·08405	·1811	·07834	·01301
19	·1212	·2611	·1130	·01876
20	·1498	·3226	·1396	·02319

Electromotive Force.

205. The electromotive force of a Daniell's cell was found by Sir W. Thomson (p. 245 of "Papers on Electricity and Magnetism") to be

·00374 electrostatic unit,

from observation of the attraction between two parallel discs connected with the opposite poles of a Daniell's battery. As 1 electrostatic unit is 3×10^{10} electromagnetic units, this is $\cdot00374 \times 3 \times 10^{10} = 1\cdot122 \times 10^{8}$ electromagnetic units, or 1·122 volt.

According to Latimer Clark's experimental determinations communicated to the Society of Telegraph Engineers in January, 1873, the electromotive force of a Daniell's cell with pure metals and saturated solutions, at 64° F., is 1·105 volt, and the electromotive force of a Grove's cell 1·97 volt. These must be diminished by 1 per cent. because they were deduced from the assumption that the B. A. unit of resistance was correct. They will thus be reduced to 1·094 and 1·95 volts.

According to the determination of F. Kohlrausch ("Pogg. Ann.," vol. cxli. [1870], and Ergänz., vol. vi. [1874], p. 35) the electromotive force of a Daniell's cell is $1\cdot138 \times 10^{8}$, and that of a Grove's cell $1\cdot942 \times 10^{8}$. These must be diminished by 3 per cent., because they were deduced from the value $\cdot9717 \times 10^{9}$ for Siemens' unit which is 3 per cent. too great. They will thus be reduced to 1·104 and 1·884 volts.

H. S. Carhart ("Amer. Jour. Sci. Art.," Nov. 1884) has found the following different values for the electromotive force of a Daniell's cell according to the strength of the zinc sulphate solution :—

Per cent. of $ZnSO_4$.	Electromotive force in volts.	Per cent.	Electromotive force.
1	1·125	10	1·118
3	1·133	15	1·115
5	1·142	20	1·111
7½	1·120	25	1·111

He finds by the same method the electromotive force of Latimer Clark's standard cell to be 1·434 volt.

Lord Rayleigh ("Phil. Trans.," June 1884, p. 452) has determined the electromotive force of a Clark cell at 15° C. to be

1·435 volt.

The value formerly assigned to it was 1·457 volt, and was based on the assumption that the B. A. unit of resistance was correct.

In a supplementary paper (Jan. 21, 1886) he gives the general result for any temperature $t°$,

$$1{\cdot}435\{1 - 0{\cdot}00077(t - 15)\},$$

together with full particulars as to the precautions necessary for securing constancy.

206. Professors Ayrton and Perry have made determinations of the electromotive forces called out by the contacts, two and two, of a great number of substances measured inductively. The method of experimenting is described in the Proceedings of the Royal Society for March 21, 1878. The following abstract of their latest results was specially prepared for this work by Professor Ayrton in January, 1879:—

CONTACT DIFFERENCES OF POTENTIAL IN VOLTS.

Solids with Solids in Air.

	Carbon.	Copper.	Iron.	Lead.	Platinum.	Tin.	Zinc.	Amalgamated Zinc.	Brass.	Average Temperature at the time of experimenting, 18° C. about.
Carbon, ...	0	·370	·485*	·858	·113	·795	1·096	1·208*	·414*	
Copper, ...	– ·370	0	·146	·542	– ·238	·456	·750	·894	·087	
Iron, ...	– ·485*	– ·146	0	·401*	– ·369	·313*	·600*	·744*	– ·064	
Lead, ...	– ·858	– ·542	– ·401*	0	– ·771	– ·099	·210	·357*	– ·472	
Platinum, ...	– ·113*	·238	·369	·771	0	·690	·981	1·125*	·287	
Tin, ...	– ·795*	– ·456	– ·313*	·099	– ·690	0	·281	·463	– ·372	
Zinc, ...	–1·096*	– ·750	– ·600*	– ·210	– ·981	– ·281	0	·144	– ·679	
Amalgamated Zinc, ...	–1·208*	– ·894	– ·744*	– ·357*	–1·125*	– ·463	– ·144	0	– ·822*	
Brass, ...	– ·414	– ·087	·064	·472	– ·287	·372	·679	·822	0	

The numbers without an asterisk were obtained directly by experiment, those with an asterisk by calculation, using the well-known assumption that in a compound circuit of metals, all at the same temperature, there is no electromotive force.

The numbers in a vertical column below the name of a substance are the differences of potential, in volts, between that substance and the substance in the same horizontal row as the number, the two substances being in contact. Thus, lead is positive to copper, the electromotive force of contact being 0·542 volts.

The metals were those of commerce, and therefore only commercially pure.

CONTACT DIFFERENCES OF POTENTIAL IN VOLTS.

		Carbon.	Copper.	Iron.	Lead.	Platinum.	Tin.
	Mercury,	·092	·308	·502	..	·156	
	Distilled water,	·01 to ·17 depending on carbon.	·269 to ·100	·148	·171	·285 to ·345	·177
SOLUTIONS.	Alum, saturated at 16°·5 C.,	..	−·127	−·653	−·139	·246	−·225
	Copper sulphate solution, specific gravity, 1·087 at 16°·6 C.,	..	·103	..	..	..	..
	Copper sulphate, saturated at 15° C.,	..	·070	..	..	..	..
	Sea salt, specific gravity, 1·18 at 20°·5 C.,	..	−·475	−·605	−·267	−·856	−·334
	Sal-ammoniac, saturated at 15°·5 C.,	..	−·396	−·652	−·189	·057	−·364
	Zinc sulphate solution, specific gravity, 1·125 at 16°·9 C.,	..	..	..	..	..	..
	Zinc sulphate, saturated at 15°·3 C.,	..	..	..	..	..	..
	1 Distilled water mixed with 3 zinc sulphate, saturated solution,	..	..	..	..	..	..
DILUTE SULPHURIC ACID.	20 Distilled water, 1 strong sulphuric acid, by weight.	..	..	..	..	..	..
	10 Distilled water, 1 strong sulphuric acid, by vol.	about −·035	..	..	..	..	..
	5 Distilled water, 1 strong sulphuric acid, by weight.	..	..	..	..	..	..
	1 Distilled water, 5 strong sulphuric acid, by weight.	·3 to ·01	..	..	−·120	..	−·25
CONCENTRATED.	Sulphuric acid,	·85 to ·55 depending on carbon.	1·113	..	·720 to 1·252	1·600 to 1·300	..
	Nitric acid,	..	..	..	..	·672	
	Mercurous sulphate paste,	..	..	..	..	..	..
	Distilled water, with a trace of sulphuric acid,	..	..	..	..	..	..

The average temperature at the time of experimenting was about 16° C.
All the liquids and salts employed were chemically pure; the solids, however, were only commercially pure.

Solids with Liquids and Liquids with Liquids in Air.

Zinc.	Amalgamated Zinc.	Brass.	Mercury.	Distilled Water.	Alum Solution, saturated at 16°·5 C.	Copper Sulphate Solution, saturated at 15° C.	Zinc Sulphate Solution, Specific Gravity 1·125 at 16°·9 C.	Zinc Sulphate Solution, saturated at 15°·3 C.	1 Distilled Water, 3 Zinc Sulphate.	Strong Nitric Acid.
−·105 to +·156	·100	·231	..	..	..	−·043	..	·164		
−·536	..	−·014								
..	..	..	..	..	..	..	·090			
..	..	..	..	−·043	..	..	..	·095	·102	
−·565	..	−·435								
−·637	..	−·348								
−·238										
−·430	−·284	..	..	−·200	..	−·095				
−·444	..	..	..	..	..	−·102				
−·344										
..	−·358									
..	−·429									
..	..	·016								
..	·848	..	..	1·298	1·456	1·269	..	1·699		
..	..	..	·475							
−·241	..	..	..	..	..	..	..	..	..	·078

Example of the above table:—Lead is positive to distilled water, and the contact difference of potentials is 0·171 volt.

The authors point out that in all these experiments the unknown electromotive forces of certain air contacts are included.

From these tables we find we can build up the electromotive forces of some well-known cells. For example, in a Daniell's cell there are four contact differences of potential to consider, and in a Grove's cell five, viz.:—

Daniell's Cell.

	Volts.
Copper and saturated copper sulphate, ...	+0·070
Saturated copper sulphate and saturated zinc sulphate,	−0·095
Saturated zinc sulphate and zinc,.....	+0·430
Zinc and copper, ...	+0·750
	1·155

Grove's Cell.

Copper and platinum, ...	+0·238
Platinum and strong nitric acid, ...	+0·672
Strong nitric acid and very weak sulphuric acid,.....	+0·078
Very weak sulphuric acid and zinc,.....	+0·241
Zinc and copper, ...	+0·750
	1·979

Thermoelectricity.

207. The electromotive force of a thermoelectric circuit is called *Thermoelectric force.* It is proportional *cæt. par.* to the number of couples. The thermoelectric force of a single couple is in the majority of cases equal to the product of two factors, one being the difference of temperature of the two junctions, and the other the difference of the *thermoelectric heights* of the two metals at a temperature midway between those of the junctions. The current through the hot junction is from the lower to the higher metal when their heights are measured at the mean temperature.

Our convention as to sign (that is, as to *up* and *down* in speaking of thermoelectric height) is the same as that adopted by Prof. Tait, and is opposite to that adopted in the first edition of this work. We have adopted it because it leads to the rule (for the Peltier and Thomson effects) that a current running down generates heat, and a current running up consumes heat.

The following table of thermoelectric heights relative to lead can be employed when the mean temperature of the two junctions does not differ much from 19° or 20° C. It is taken from Jenkin's "Electricity and Magnetism," p. 176, where it is described as being compiled from Matthiessen's experiments. We have reversed the signs to suit the above convention, and have multiplied by 100 to reduce from microvolts to C.G.S. units.

Thermoelectric Heights at about 20° C.

Bismuth, pressed commercial wire,	−9700	Antimony, pressed wire	+280
Bismuth, pure pressed wire,	−8900	Silver, pure hard,	+300
Bismuth, crystal, axial,	−6500	Zinc, pure pressed,	+370
,, equatorial,	−4500	Copper, galvano-plastically precipitated,	+380
Cobalt,	−2200	Antimony, pressed commercial wire,	+600
German Silver,	−1175	Arsenic,	+1356
Quicksilver,	−41·8	Iron, pianoforte wire,	+1750
Lead,	0	Antimony, axial,	+2260
Tin,	+10	,, equatorial,	+2640
Copper of Commerce,	+10	Phosphorus, red,	+2970
Platinum,	+90	Tellurium,	+50200
Gold,	+120	Selenium,	+80700

208. The following table is based upon Professor Tait's thermoelectric diagram ("Trans. Roy. Soc., Edin.," vol. xxvii. 1873) joined with the assumption that a Grove's cell has electromotive force $1{\cdot}97 \times 10^8$:—

	Thermoelectric Heights at $t°$ C. in C.G.S. units.
Iron,	$+1734 - 4{\cdot}87\,t$
Steel,	$+1139 - 3{\cdot}28\,t$
Alloy, believed to be Platinum Iridium,	$+ 839$ at all temperatures.
Alloy, Platinum 95; Iridium 5,	$+ 622 - {\cdot}55\,t$
,, ,, 90; ,, 10,	$+ 596 - 1{\cdot}34\,t$
,, ,, 85; ,, 15,	$+ 709 - {\cdot}63\,t$
,, ,, 85; ,, 15,	$+ 577$ at all temperatures.
Soft Platinum,	$- 61 - 1{\cdot}10\,t$
Alloy, platinum and nickel,	$+ 544 - 1{\cdot}10\,t$
Hard Platinum,	$+ 260 - {\cdot}75\,t$
Magnesium,	$+ 244 - {\cdot}95\,t$
German Silver,	$-1207 - 5{\cdot}12\,t$
Cadmium,	$+ 266 + 4{\cdot}29\,t$
Zinc,	$+ 234 + 2{\cdot}40\,t$
Silver,	$+ 214 + 1{\cdot}50\,t$
Gold,	$+ 283 + 1{\cdot}02\,t$
Copper,	$+ 136 + {\cdot}95\,t$
Lead,	0
Tin,	$- 43 + {\cdot}55\,t$
Aluminium,	$- 77 + {\cdot}39\,t$
Palladium,	$- 625 - 3{\cdot}59\,t$
Nickel to 175° C.,	$-2204 - 5{\cdot}12\,t$
,, 250° to 310° C.,	$-8449 + 24{\cdot}1\,t$
,, from 340° C.,	$- 307 - 5{\cdot}12\,t$

The lower limit of temperature for the table is $-18°$ C. for all the metals in the list. The upper limit is 416° C., with the following exceptions:—Cadmium, 258° C.; Zinc, 373° C.; German Silver, 175° C.

Ex. 1. Required the electromotive force of a copper-iron couple, the temperatures of the junctions being 0° C. and 100° C.

We have, for iron, $+ 1734 - 4{\cdot}87t$;

,, copper, $+ 136 + {\cdot}95t$;

,, iron above copper, $1598 - 5{\cdot}82t$.

The electromotive force per degree is

$$1598 - 5{\cdot}82 \times 50 = 1307,$$

and the electromotive force of the couple is

$$1307(100 - 0) = 130{,}700,$$

tending from copper to iron through the hot junction.

By the *neutral point* of two metals is meant the temperature at which their thermoelectric heights are equal.

Ex. 2. To find the neutral point of copper and iron we have

$$1598 - 5{\cdot}82t = 0,\ t = 275\ ;$$

that is, the neutral point is 275° C. When the mean of the temperatures of the junctions is below this point, the current through the warmer junction is from copper to iron. The current ceases as the mean temperature attains the neutral point, and is reversed in passing it.

Ex. 3. F. Kohlrausch ("Pogg. Ann. Ergänz.," vol. vi. p. 35, 1874) states that, according to his determination, the electromotive force of a couple of iron and German silver is 24×10^5 millimetre-milligramme-second units for 1° of difference of temperatures of the junctions, at moderate temperatures. Compare this result with the above Table at mean temperature 100°.

The dimensions of electromotive force are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$; hence the C.G.S. value of Kohlrausch's unit is $10^{-\frac{3}{2}}10^{-\frac{3}{2}} = 10^{-3}$, giving 2400 as the electromotive force per degree of difference.

From the above table we have

$$\text{Iron above German silver, } 2941 + {\cdot}25t,$$

which, for $t = 100$, gives 2966 as the electromotive force per degree of difference.

Peltier and Thomson Effects.

209. When a current is sent through a circuit composed of different metals, it produces in general three distinct thermal effects.

1. A generation of heat to the amount per second of C^2R ergs, C denoting the current, and R the resistance.

2. A generation of heat or cold at the junctions. This is called the Peltier effect, and its amount per second in ergs at any one junction can be computed by multiplying the difference of thermoelectric heights at this junction by $t + 273$ and by the current, t denoting the centigrade temperature of the junction. If the current flows down (that is from greater to less thermoelectric height) the effect is a warming; if it flows up, the effect is a cooling.

Ex. 4. Let a unit current (or a current of 10 amperes) flow through a junction of copper and iron at 100° C.

The thermoelectric heights at 100° C. are

Iron,	1247
Copper,	231
Iron above copper,	1016

Multiplying 1016 by 373, we have about 379,000 ergs, or $\frac{1}{111}$ of a gramme-degree, as the Peltier effect per second. Heat of this amount will be generated if the current is from iron to copper, and will be destroyed if the current is from copper to iron.

3. A generation of heat or cold in portions of the circuit consisting of a single metal in which the temperature varies from point to point. This is called the Thomson effect. Its amount per second, for any such portion of

the circuit, is the difference of the thermoelectric heights of the two ends of the portion, multiplied by $273 + t$, where t denotes the half-sum of the centigrade temperatures of the ends, and by the strength of the current.

The Thomson effect, like the Peltier effect, is reversed by reversing the current, and follows the same rule that *heat is generated* when the current is from *greater to less thermoelectric height.*

Experiment shows that the Thomson effect is insensible in the case of lead; hence the thermoelectric height of lead must be sensibly the same at all temperatures. It is for this reason that lead is adopted, by common consent, as the zero from which thermoelectric heights are to be reckoned.

Ex. 5. In an iron wire with ends at 0° C. and 100° C., the cold end is the higher (thermoelectrically) by $4{\cdot}87 \times 100$—that is, by 487. Multiplying this difference by $273 + \frac{1}{2}(0 + 100)$ or 323, we have 157300 as the Thomson effect per second for unit current. This amount of heat (in ergs) is generated in the iron when the current through it is from the cold to the hot end, and is destroyed when the current is from hot to cold.

Ex. 6. In a copper wire with ends at 0° C. and 100° C., the hot end is the higher by $\cdot 95 \times 100$ or 95. Multiplying this by 323, we have 30700 (ergs) as the Thomson effect per second per unit current. This amount of heat is generated in the copper when the current through it is from hot to cold, and destroyed when the current is from cold to hot.

The effect of a current from hot to cold is opposite in these two metals, because the coefficients of t in the

expressions for their thermoelectric heights (p. 174) have opposite signs.

Relation between Thermoelectric Force and the Peltier and Thomson effects.

210. The algebraic sum of the Peltier and Thomson effects (expressed in ergs) due to unit current for one second in a closed metallic circuit, is equal to the thermoelectric force of the circuit; and the direction of this thermoelectric force is the direction of a current round the circuit which would give an excess of destruction over generation of heat (so far as these two effects are concerned).

Ex. 7. In a copper-iron couple with junctions at 0° C. and 100° C., suppose a unit current to circulate in such a direction as to pass from copper to iron through the hot junction, and from iron to copper through the cold junction.

The Peltier effect at the hot junction is a destruction of heat to the amount $1016 \times 373 = 379{,}000$ ergs.

The Peltier effect at the cold junction is a generation of heat to the amount $1598 \times 273 = 436{,}300$ ergs.

The Thomson effect in the iron is a destruction of heat to the amount $487 \times 323 = 157{,}300$ ergs.

The Thomson effect in the copper is a destruction of heat to the amount $95 \times 323 = 30{,}700$ ergs.

The total amount of destruction is 567,000, and of generation 436,300, giving upon the whole a destruction of 130,700 ergs. The electromotive force of the couple is therefore 130,700, and tends in the direction of the current here supposed. This agrees with the calculation in Example 1.

Electrochemical Equivalents.

211. The quantity of a given metal deposited in an electrolytic cell or dissolved in a battery cell (when there is no "local action") depends on the quantity of electricity that passes, irrespective of the time occupied. Hence we can speak definitely of the quantity of the metal that is "equivalent to" a given quantity of electricity. By the *electrochemical equivalent* of a metal is meant the quantity of it that is equivalent to the unit quantity of electricity. In the C. G. S. system it is the number of grammes of the metal that are equivalent to the C.G.S. electromagnetic unit of electricity.

Special attention has been paid to the electrochemical equivalent of silver, as this metal affords special facilities for accurate measurement of the deposit. The latest experiments of Lord Rayleigh and Kohlrausch agree in giving

$$\cdot 01118$$

as the C.G.S. electrochemical equivalent of silver.*

The number of grammes of silver deposited by 1 ampere in one hour is

$$\cdot 01118 \times \tfrac{1}{10} \times 3600 = 4 \cdot 025.$$

212. The electrochemical equivalents of the most important of the elements are given in the following table. They are calculated from the chemical equivalents in the preceding column by simple proportion, taking as basis the above-named value for silver. Their reciprocals are the quantities of electricity required for depositing one

* Rayleigh's determination is ·0111794; Kohlrausch's, ·011183; Mascart's, ·011156. See "Phil. Trans.," 1884, pp. 439, 458.

gramme. The quantity of electricity required for depositing the number of grammes stated in the column "chemical equivalents" is the same for all the elements, namely, 9634 C.G.S. units.

Elements.	Atomic Weight.	Valency.	Chemical Equivalents.	Electrochemical equivalents or grammes per unit of electricity.	Reciprocal or Electricity per gramme.
Electro-positive—					
Hydrogen,	1	1	1	·0001038	9634
Potassium,	39·03	1	39·03	·004051	246·9
Sodium,	23·00	1	23·00	·002387	418·9
Gold,	196·2	3	65·4	·006789	147·3
Silver,	107·7	1	107·7	·01118	89·45
Copper (cupric),	63·18	2	31·59	·003279	305·0
,, (cuprous),		1	63·18	·006558	152·5
Mercury (mercuric),	199·8	2	99·9	·01037	96·43
,, (mercurous),		1	199·8	·02074	48·22
Tin (stannic),	117·4	4	29·35	·003046	328·3
,, (stannous),		2	58·7	·006093	164·1
Iron (ferric),	55·88	3	18·63	·001934	517·1
,, (ferrous),		2	27·94	·002900	344·8
Nickel,	58·6	2	29·3	·003042	328·7
Zinc,	64·88	2	32·44	·003367	297·0
Lead,	206·4	2	103·2	·01071	93·37
Aluminium,	27·04	3	9·01	·000935	1070
Electro-negative—					
Oxygen,	15·96	2	7·98	·0008283	1207
Chlorine,	35·37	1	35·37	·003671	272·4
Iodine,	126·54	1	126·54	·013134	76·14
Bromine,	79·76	1	79·76	·008279	120·8
Nitrogen,	14·01	3	4·67	·0004847	2063

To find the equivalent of 1 coulomb, divide the above electrochemical equivalents by 10.

To find the number of grammes deposited per hour by 1 ampere, multiply the above electrochemical equivalents by 360.

213. Let the "chemical equivalents" in the above table be taken as so many grammes; then, if we denote by H the amount of heat due to the whole chemical action which takes place in a battery cell during the consumption of one equivalent of zinc, the chemical energy which runs down, namely JH ergs, must be equal (if there is no wasteful local action) to the energy of the current produced. But this is the product of the quantity of electricity 9634 by the electromotive force of the cell. The electromotive force is therefore equal to $\frac{JH}{9634}$.

In the tables of heats of combination which are in use among chemists, the equivalent of hydrogen is taken as 2 grammes, and that of zinc as 64·88 or 65 grammes. The equivalent quantity of electricity will accordingly be 9634×2, and the formula to be used for calculating the electromotive force of a cell will be $\frac{JH}{19268}$.

In applying this calculation to Daniell's and Grove's cells, we shall employ the following heats of combination, which are given on page 614 of Watts' "Dictionary of Chemistry," vol. vii., and are based on Julius Thomsen's observations:—

Zn, O, SO^3, Aq., ...	108,462
Cu, O, SO^3, Aq., ...	54,225
N^2O^2, O^3, Aq., ...	72,940
N^2O^2, O, Aq., ...	36,340

In Daniell's cell, zinc is dissolved and copper is set free, we have, accordingly,

$$H = 108,462 - 54,225 = 54,237.$$

In Grove's cell, zinc is dissolved and nitric acid is

changed into nitrous acid. The thermal value of this latter change can be computed from the third and fourth data in the above list, as follows:—

72,940 is the thermal value of the action in which, by the oxidation of one equivalent of N^2O^2 and combination with water, two equivalents of NHO^3 (nitric acid) are produced.

36,340 is the thermal value of the action in which, by the oxidation of one equivalent of N^2O^2 and combination with water, two equivalents of NHO^2 (nitrous acid) are produced. The difference 36,600 is accordingly the thermal value of the conversion of two equivalents of nitrous into nitric acid, and 18,300 is the value for the conversion of one equivalent. In the present case the reverse changes take place. We have, therefore,

$$H = 108{,}462 - 18{,}300 = 90{,}162.$$

Taking J as $4{\cdot}2 \times 10^7$, the value of $\frac{JH}{19268}$ will be

$1{\cdot}182 \times 10^8$ for Daniell's cell.

$1{\cdot}965 \times 10^8$ „ Grove's „

These are greater by from 2 to 8 per cent. than the direct determinations given in § 205.

214. *Examples in Electricity.*

1. Two conducting spheres, each of 1 centim. radius, are placed at a distance of r centims. from centre to centre, r being a large number; and each of them is charged with an electrostatic unit of positive electricity. With what force will they repel each other?

Since r is large, the charge may be assumed to be uniformly distributed over their surfaces, and the force will be the same as if the charge of each were collected at its centre. The force will therefore be $\frac{1}{r^2}$ of a dyne.

2. Two conducting spheres, each of 1 centim. radius, placed as in the preceding question, are connected one with each pole of a Daniell's battery (the middle of the battery being to earth) by means of two very fine wires whose capacity may be neglected, so that the capacity of each sphere when thus connected is sensibly equal to unity. Of how many cells must the battery consist that the spheres may attract each other with a force of $\frac{1}{r^2}$ of a dyne, r being the distance between their centres in centims.?

One sphere must be charged to potential 1 and the other to potential -1. The number of cells required is

$$\frac{2}{\cdot 00374} = 535.$$

3. How many Daniell's cells would be required to produce a spark between two parallel conducting surfaces at a distance of ·019 of a centim., and how many at a distance of ·0086 of a centim.? (See §§ 178, 184.)

Ans. $\frac{4 \cdot 26}{\cdot 00374} = 1139$; $\frac{2 \cdot 30}{\cdot 00374} = 615.$

4. Compare the capacity denoted by 1 farad with the capacity of the earth.

The capacity of the earth in static measure is equal to its radius, namely $6 \cdot 37 \times 10^8$. Dividing by v^2 to reduce to magnetic measure, we have $\cdot 71 \times 10^{-12}$, which is 1 farad multiplied by $\cdot 71 \times 10^{-3}$, or is ·00071 of a farad. A farad is therefore 1400 times the capacity of the earth.

5. Calculate the resistance of a cell consisting of a plate of zinc, A square centims. in area, and a plate of copper of the same dimensions, separated by an acid

solution of specific resistance 10^9, the distance between the plates being 1 centim.

Ans. $\frac{10^9}{A}$, or $\frac{1}{A}$ of an ohm.

6. Find the heat developed in 10 minutes by the passage of a current from 10 Daniell's cells in series through a wire of resistance 10^{10} (that is, 10 ohms), assuming the electromotive force of each cell to be $1{\cdot}1 \times 10^8$, and the resistance of each cell to be 10^9.

Here we have

Total electromotive force $= 1{\cdot}1 \times 1^9$.

Resistance in battery $= 10^{10}$.

Resistance in wire $= 10^{10}$.

Current $= \frac{1{\cdot}1 \times 10^9}{2 \times 10^{10}} = {\cdot}55 \times 10^{-1} = {\cdot}055$.

Heat developed in wire per second $= \frac{({\cdot}055^2) \times 10^{10}}{4{\cdot}2 \times 10^7} = 7{\cdot}2024$.

Hence the heat developed in 10 minutes is 4321·4 gramme-degrees.

7. Find the electromotive force between the wheels on opposite sides of a railway carriage travelling at the rate of 30 miles an hour on a line of the ordinary gauge [4 feet $8\frac{1}{2}$ inches] due to cutting the lines of force of terrestrial magnetism, the vertical intensity being ·438.

The electromotive force will be the product of the velocity of travelling, the distance between the rails, and the vertical intensity, that is,

$$(44{\cdot}7 \times 30)\ (2{\cdot}54 \times 56{\cdot}5)\ ({\cdot}438) = 84{,}300$$

electromagnetic units.

This is about $\frac{1}{1200}$ of a volt.

8. Find the electromotive force at the instant of passing the magnetic meridian, in a circular coil consisting of 300 turns of wire, revolving at the rate of 10 revolutions per second about a vertical diameter; the diameter of the coil being 30 centims., and the horizontal intensity of terrestrial magnetism being ·1794, no other magnetic influence being supposed present.

Self-induction can be left out of account, because the current is a maximum.

The numerical value of the lines of force which go through the coil when inclined at an angle θ to the meridian, is the horizontal intensity multiplied by the area of the coil and by sin θ; say $n\mathrm{H}\pi a^2 \sin\theta$, where $\mathrm{H} = ·1794$, $a = 15$, and $n = 300$. The electromotive force at any instant is the rate at which this quantity increases or diminishes; that is, $n\mathrm{H}\pi a^2 \cos\theta \,.\, \omega$, if ω denote the angular velocity. At the instant of passing the meridian cos θ is 1, and the electromotive force is $n\mathrm{H}\pi a^2\omega$. With 10 revolutions per second the value of ω is $2\pi \times 10$.

Hence the electromotive force is

$$·1794 \times (3·142)^2 \times 225 \times 20 \times 300 = 2·39 \times 10^6.$$

This is about $\frac{1}{42}$ of a volt.

190. To investigate the magnitudes of units of length, mass, and time which will fulfil the three following conditions:—

1. The acceleration due to the attraction of unit mass at unit distance shall be unity.

2. The electrostatic units shall be equal to the electromagnetic units.

3. The density of water at 4° C. shall be unity.

Let the 3 units required be equal respectively to L centims., M grammes, and T seconds.

We have in C.G.S. measure, for the acceleration due to attraction (§ 72),

$$\text{acceleration} = C\,\frac{\text{mass}}{(\text{distance})^2}, \text{ where } C = 6{\cdot}48 \times 10^{-8};$$

and in the new system we are to have

$$\text{acceleration} = \frac{\text{mass}}{(\text{distance})^2}.$$

Hence, by division,

$$\frac{\text{acceleration in C.G.S. units}}{\text{acceleration in new units}} = C\,\frac{\text{mass in C.G.S. units}}{\text{mass in new units}} \cdot \frac{(\text{distance in new units})^2}{(\text{distance in C.G.S. units})^2};$$

that is, $\frac{L}{T^2} = C\frac{M}{L^2}$.

This equation expresses the first of the three conditions.

The equation $\frac{L}{T} = v$ expresses the second, v denoting 3×10^{10}.

The equation $M = L^3$ expresses the third.

Substituting L^3 for M in the first equation, we find $T = \sqrt{\frac{1}{C}}$. Hence, from the second equation,

$$L = v\sqrt{\frac{1}{C}};$$

and from the third,

$$M = \left(v\sqrt{\frac{1}{C}}\right)^3.$$

Introducing the actual values of C and v, we have approximately

$$T = 3928,\ L = 1{\cdot}178 \times 10^{14},\ M = 1{\cdot}63 \times 10^{42};$$

that is to say,

The new unit of time will be about $1^h\ 5\frac{1}{2}^m$;

The new unit of length will be about 118 thousand earth quadrants;

The new unit of mass will be about $2{\cdot}66 \times 10^{14}$ times the earth's mass.

Electrodynamics.

191. Ampère's formula for the repulsion between two elements of currents, when expressed in electromagnetic units, is

$$\frac{cc'\ ds\ .\ ds'}{r^2}(2 \sin\alpha \sin\alpha' \cos\theta - \cos\alpha \cos\alpha'),$$

where c, c' denote the strengths of the two currents;

ds, ds' the lengths of the two elements;

α, α' the angles which the elements make with the line joining them;

r the length of this joining line;

θ the angle between the plane of r, ds, and the plane of r, ds'.

For two parallel currents, one of which is of infinite length, and the other of length l, the formula gives by integration an attraction or repulsion,

$$\frac{2l}{D}cc',$$

where D denotes the perpendicular distance between the currents.

Example.

Find the attraction between two parallel wires a metre long and a centim. apart when a current of $\frac{1}{10}$ is passing through each.

Here the attraction will be sensibly the same as if one of the wires were indefinitely increased in length, and will be

$$\frac{200}{1}\left(\frac{1}{10}\right)^2 = 2\,;$$

that is, each wire will be attracted or repelled with a force of 2 dynes, according as the directions of the currents are the same or opposite.

OMISSION (TO BE ADDED TO § 63, p. 61).

ACCORDING to experiments by Quincke (Berlin Transactions, April 5, 1885) the following are the compressions due to the pressure of one atmosphere. They are expressed in millionths of the original volume :—

	Compression in millionths.		
	at 0° C.	at t° C.	t.
Glycerine,	25·24	25·10	19·00
Rape oil (rüböl),	48·02	58·18	17·80
Almond oil,	48·21	56·30	19·68
Olive oil,	48·59	61·74	18·3
Water,	50·30	45·63	22·93
Bisulphide of carbon,	53·92	63·78	17·00
Oil of turpentine,	58·17	77·93	18·56
Benzol from benzoic acid,	—	66·10	16·78
Benzol,	—	62·84	16·08
Petroleum,	64·99	74·50	19·23
Alcohol,	82·82	95·95	17·51
Ether,	115·57	147·72	21·36

CORRECTION (p. 84).

BENOÎT'S results on refraction of air will not appear in vol. v., but in a later volume.

SUGGESTION FOR WRITING DECIMAL MULTIPLES AND SUBMULTIPLES.

PROFESSOR NEWCOMB has suggested, as a possible improvement in future editions of this work, the employment of powers of 1000 instead of powers of 10 as factors (a plan which corresponds with the usual division of digits into periods of 3 each), and the employment of the letter m in this connection to denote 1000.

Thus, instead of $1{\cdot}226 \times 10^5$, we should write $122{\cdot}6\ m$.

$1{\cdot}006 \times 10^7$, „ $10{\cdot}06\ m^2$.

$\cdot 000\ 000\ 9$, „ $\cdot 9\ m^{-2}$.

The plan appears to possess some advantages; and if the symbol m for 1000 is not sufficiently self-explanatory, we might write $122{\cdot}6 \times 10^3$, $10{\cdot}06 \times 10^6$, $\cdot 9 \times 10^{-6}$. We place the suggestion on record here that it may not be overlooked.

UNIVERSITY OF CALIFORNIA
DEPARTMENT OF PHYSICS

APPENDIX.

First Report of the Committee for the Selection and Nomenclature of Dynamical and Electrical Units, the Committee consisting of SIR W. THOMSON, F.R.S., PROFESSOR G. C. FOSTER, F.R.S., PROFESSOR J. C. MAXWELL, F.R.S., MR. G. J. STONEY, F.R.S.,* PROFESSOR FLEEMING JENKIN, F.R.S., DR. SIEMENS, F.R.S., MR. F. J. BRAMWELL, F.R.S., *and* PROFESSOR EVERETT (Reporter).

WE consider that the most urgent portion of the task intrusted to us is that which concerns the selection and nomenclature of units of force and energy; and under this head we are prepared to offer a definite recommendation.

A more extensive and difficult part of our duty is the selection and nomenclature of electrical and magnetic units. Under this head we are prepared with a definite recommendation as regards selection, but with only an interim recommendation as regards nomenclature.

Up to the present time it has been necessary for every person who wishes to specify a magnitude in what is called "absolute" measure, to mention the three fundamental units of mass, length, and time which he has chosen as the basis of his system. This necessity will be obviated if one definite selection of three fundamental units be made once for all, and accepted by the general consent of scientific men. We are strongly of opinion that such a selection ought at once to be made, and to be so made that there will be no subsequent necessity for amending it.

We think that, in the selection of each kind of derived unit, all arbitrary multiplications and divisions by powers of ten, or other factors, must be rigorously avoided, and the whole system of

* Mr. Stoney objected to the selection of the centimetre as the unit of length.

fundamental units of force, work, electrostatic, and electromagnetic elements must be fixed at one common level—that level, namely, which is determined by direct derivation from the three fundamental units once for all selected.

The carrying out of this resolution involves the adoption of some units which are excessively large or excessively small in comparison with the magnitudes which occur in practice; but a remedy for this inconvenience is provided by a method of denoting decimal multiples and sub-multiples, which has already been extensively adopted, and which we desire to recommend for general use.

On the initial question of the particular units of mass, length, and time to be recommended as the basis of the whole system, a protracted discussion has been carried on, the principal point discussed being the claims of the gramme, the *metre*, and the second, as against the gramme, the *centimetre*, and the second,—the former combination having an advantage as regards the simplicity of the name *metre*, while the latter combination has the advantage of making the unit of mass practically identical with the mass of unit-volume of water—in other words, of making the value of the density of water practically equal to unity. We are now all but unanimous in regarding this latter element of simplicity as the more important of the two; and in support of this view we desire to quote the authority of Sir W. Thomson, who has for a long time insisted very strongly upon the necessity of employing units which conform to this condition.

We accordingly recommend the general adoption of the *Centimetre*, the *Gramme*, and the *Second* as the three fundamental units; and until such time as special names shall be appropriated to the units of electrical and magnetic magnitude hence derived, we recommend that they be distinguished from "absolute" units otherwise derived, by the letters "C.G.S." prefixed, these being the initial letters of the names of the three fundamental units.

Special names, if short and suitable, would, in the opinion of a majority of us, be better than the provisional designations "C.G.S. unit of . . . ." Several lists of names have already been suggested; and attentive consideration will be given to any further

suggestions which we may receive from persons interested in electrical nomenclature.

The "ohm," as represented by the original standard coil, is approximately 10^9 C.G.S. units of resistance; the "volt" is approximately 10^8 C.G.S. units of electromotive force; and the "farad" is approximately $\frac{1}{10^9}$ of the C.G.S. unit of capacity.

For the expression of high decimal multiples and sub-multiples, we recommend the system introduced by Mr. Stoney, a system which has already been extensively employed for electrical purposes. It consists in denoting the exponent of the power of 10, which serves as multiplier, by an appended cardinal number, if the exponent be positive, and by a prefixed ordinal number if the exponent be negative.

Thus 10^9 grammes constitute a *gramme-nine;* $\frac{1}{10^9}$ of a gramme constitutes a *ninth-gramme;* the approximate length of a quadrant of one of the earth's meridians is a *metre-seven*, or a *centimetre-nine.*

For multiplication or division by a million, the prefixes *mega** and *micro* may conveniently be employed, according to the present custom of electricians. Thus the *megohm* is a million ohms, and the *microfarad* is the millionth part of a farad. The prefix *mega* is equivalent to the affix *six*. The prefix *micro* is equivalent to the prefix *sixth*.

The prefixes *kilo*, *hecto*, *deca*, *deci*, *centi*, *milli* can also be employed in their usual senses before all new names of units.

As regards the name to be given to the C.G.S. *unit of force*, we recommend that it be a derivative of the Greek δύναμις. The form *dynamy* appears to be the most satisfactory to etymologists. *Dynam* is equally intelligible, but awkward in sound to English ears. The shorter form, *dyne*, though not fashioned according to strict rules of etymology, will probably be generally preferred in this country. Bearing in mind that it is desirable to construct a system with a view to its becoming international, we think that

* Before a vowel, either *meg* or *megal*, as euphony may suggest, may be employed instead of *mega*.

the termination of the word should for the present be left an open question. But we would earnestly request that, whichever form of the word be employed, its meaning be strictly limited to the unit of force of the C.G.S. system—that is to say, *the force which, acting upon a gramme of matter for a second, generates a velocity of a centimetre per second.*

The C.G.S. *unit of work* is the work done by *this force working through a centimetre;* and we propose to denote it by some derivative of the Greek ἔργον. The forms *ergon, ergal,* and *erg* have been suggested; but the second of these has been used in a different sense by Clausius. In this case also we propose, for the present, to leave the termination unsettled; and we request that the word *ergon,* or *erg,* be strictly limited to the C.G.S. unit of work, or what is, for purposes of measurement, equivalent to this, the C.G.S. *unit of energy,* energy being measured by the amount of work which it represents.

The C.G.S. *unit of power* is the power of doing work at the rate of *one erg per second;* and the power of an engine, under given conditions of working, can be specified in *ergs per second.*

For rough comparison with the vulgar (and variable) units based on terrestrial gravitation, the following statement will be useful:—

The *weight* of a *gramme,* at any part of the earth's surface, is about 980 *dynes,* or rather less than a *kilodyne.*

The *weight* of a *kilogramme* is rather less than a *megadyne,* being about 980,000 *dynes.*

Conversely, the *dyne* is about 1·02 times the *weight* of a *milligramme* at any part of the earth's surface; and the *megadyne* is about 1·02 times the *weight* of a *kilogramme.*

The *kilogrammetre* is rather less than the *ergon-eight,* being about 98 million *ergs.*

The *gramme-centimetre* is rather less than the *kilerg,* being about 980 *ergs.*

For exact comparison, the value of g (the acceleration of a body falling in vacuo) at the station considered must of course be known. In the above comparison it is taken as 980 C.G.S. units of acceleration.

One *horse-power* is about three quarters of an *erg-ten* per second. More nearly, it is 7·46 *erg-nines* per second, and one *force-de-cheval* is 7·36 *erg-nines* per second.

The mechanical equivalent of one *gramme-degree* (Centigrade) of heat is 41·6 megalergs, or 41,600,000 *ergs*.

Second Report of the Committee for the Selection and Nomenclature of Dynamical and Electrical Units, the Committee consisting of Professor Sir W. Thomson, F.R.S., Professor G. C. Foster, F.R.S., Professor J. Clerk Maxwell, F.R.S., G. J. Stoney, F.R.S., Professor Fleeming Jenkin, F.R.S., Dr. C. W. Siemens, F.R.S., F. J. Bramwell, F.R.S., Professor W. G. Adams, F.R.S., Professor Balfour Stewart, F.R.S., *and* Professor Everett (Secretary).

The Committee on the Nomenclature of Dynamical and Electrical Units have circulated numerous copies of their last year's Report among scientific men both at home and abroad.

They believe, however, that, in order to render their recommendations fully available for science teaching and scientific work, a full and popular exposition of the whole subject of physical units is necessary, together with a collection of examples (tabular and otherwise) illustrating the application of systematic units to a variety of physical measurements. Students usually find peculiar difficulty in questions relating to units; and even the experienced scientific calculator is glad to have before him concrete examples with which to compare his own results, as a security against misapprehension or mistake.

Some members of the Committee have been preparing a small volume of illustrations of the C.G.S. system [Centimetre-Gramme-Second system] intended to meet this want.

[The first edition of the present work is the volume of illustrations here referred to.]

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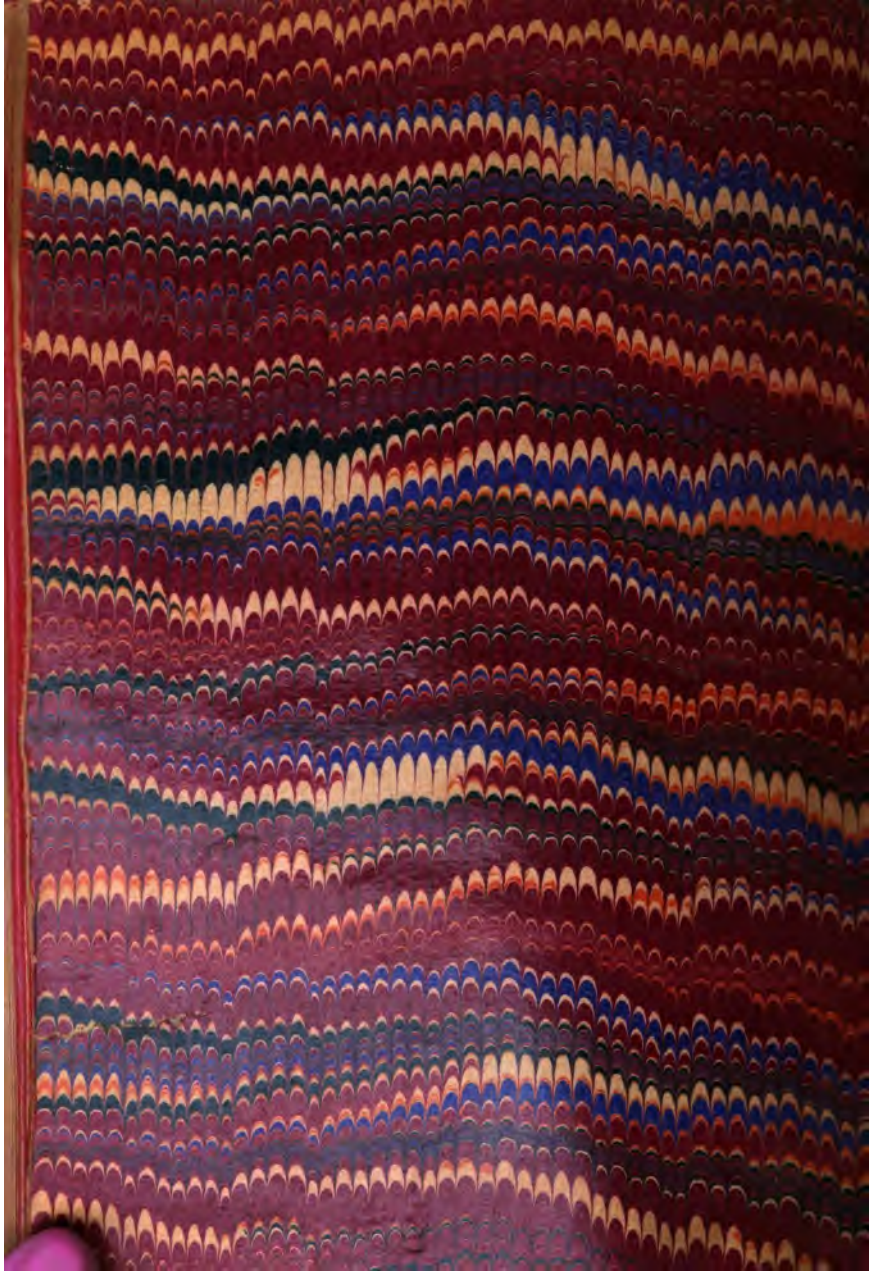

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